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Microcontroller Assignment

Q1> Explain ARM core data flow model with diagram.

Q2> Explain the architecture of Embedded system device based on ARM core with diagram

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Solution :-

Q1> Data flow model :-

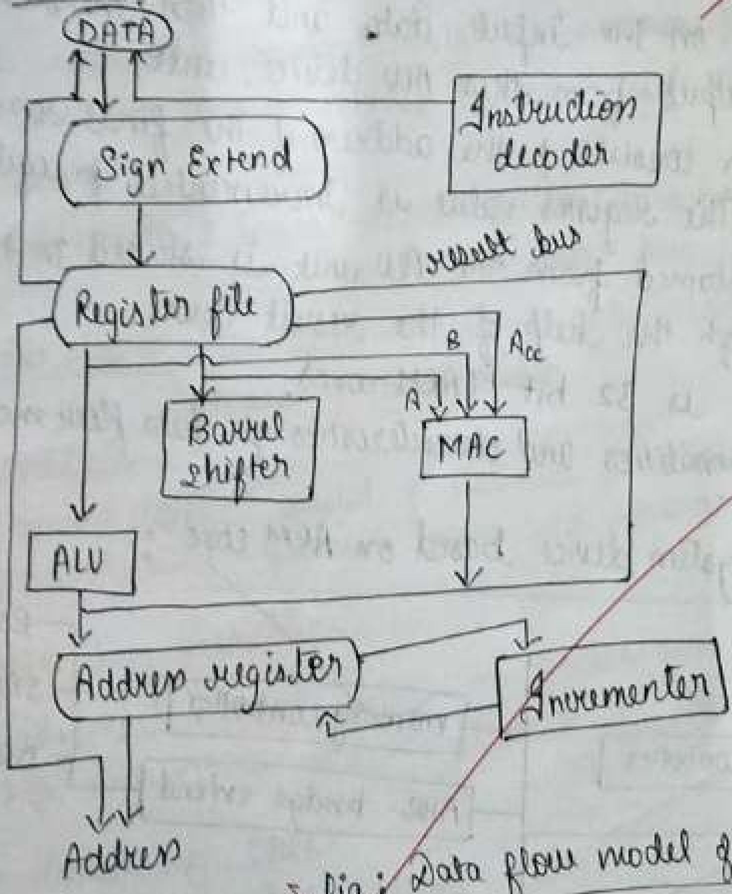


fig: Data flow model of ARM CORE

→ The arrow represent the flow of data within the system

→ The lines represent various buses which are connected from one device to another device. The data transfer between components take place through the buses.

→ The boxes represent various operations or the data storage contain data to perform the operation.

→ The instruction set are first decoded by the instruction decoder before it is executed by the processor.

→ The core contains various instruction sets.

→ The logic function load and store architecture is used.

→ Therefore there will be two instruction set required for in and out transfer of the data throughout the system.

→ The barrel shifter preprocess the register before it is executed.

→ The register file is the main storage when the input data is being stored. It consists of values and results.

→ The arithmetic logic unit is a logical device which perform logical computation on the input data and then the result is provided as output from this ALU device unit.

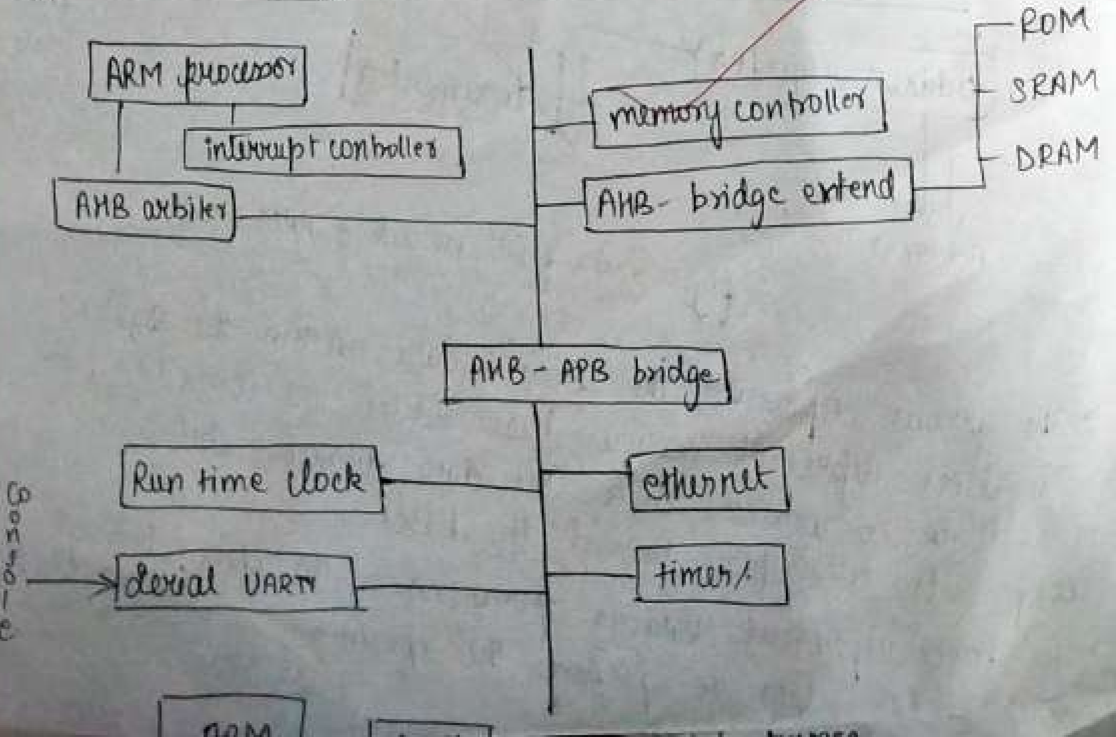
→ The address register consists of the address of the final result produced (value). The register value is incremented for cycle.

→ The result value obtained from the ALU unit is stored in the register file through the help of the result bus.

→ The processor used is 32 bit (ARM core).

→ These are the functionalities and characteristics of data flow model.

Q2> <u>Embedded System device based on ARM core</u> :-



-> The ARM processor controlls the entire embedded device.
-> The arm core is sorrounded by several components of system.
-> It consists of two mainly function components. They are the memory controller and the interrupt controller.
-> The peripherals main function is input / output functions which are off chip and it exibit uniqueness of the embedded system.

1> <u>ARM bus technology</u> :

-> Embedded devices consist of various bus technology set.
-> Out of peripheral counter (PC), the peripheral counter interconnect (PCI) is an off chip devic bus.
-> The embedded system consist of various peripherals which are used to connect various devices through the help of off chip.

two bus :

(i) <u>Bus master</u> : logical device which transfer data to another device across the same bus

(ii) <u>Bus slave</u> : logical device which only respond to the transfer request of bus master

two architecture :

(i) Physical level - consist of electrical charecteristic and bit width (8,16,32)
(ii) Secondary level - protocol - rules and regulation that govern the system.

2> <u>AMBA bus architecture</u> :-

- The advanced microcontroller bus architecture is widely used ON-chip architecture which is used in ARM processor.

(i) ARM system bus (ASB)
(ii) ~~APB~~ ARM peripheral bus (APB)
(iii) ARM high performance bus (AHB).

-> plug and play interface for developers :-
-> The design can be reused and apply for various multiprojects.
-> It is just to connect to the chip and not required to interface new for various devices.
-> High clock speed
->

3) Memory :-

→ There is requirement of a memory storage space for storing code for execute.

we have ;

cache : It is fast memory. Small storage space. it is situated between the CORE and main memory.

main memory : Slower compared to cache. It is used to store more codes to be executed.

secondary storage : It is slower than rest of memory devices.
- It has large storage space.
- All the larger data is stored in this secondary storage.

4) Peripherals :-

→ Outside world external execution. connection of embedded system (off chip)

→ Main purpose is the input / output function. transfer of data through devices

→ It can execute one function at a time.

(i) Memory controller

(ii) Interrupt controller.

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Assignment-3

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1) A 3-ϕ generator with an open ckt voltage of 400V is subjected to an LG fault through a fault impedance of $j2\Omega$. Determine the fault current if $Z_1 = j4\Omega$, $Z_2 = j2\Omega$ & $Z_0 = j1\Omega$. Repeat the problem for LL & LLG fault.

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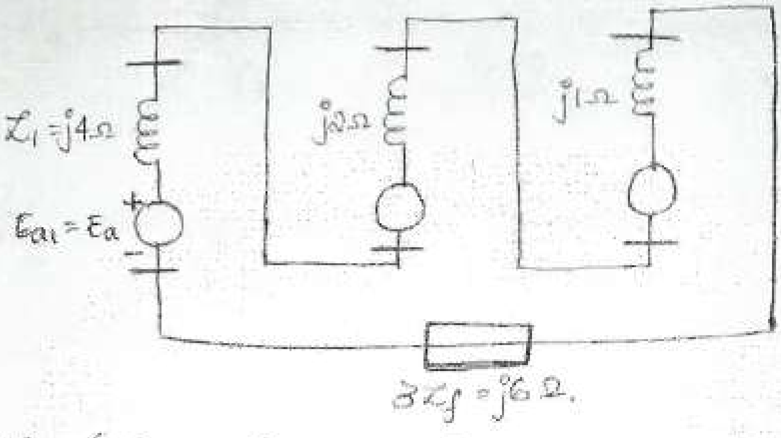
Ans (i) LG fault :-

The interconnection of sequence network for an LG fault is shown.



in this case, $I_{a1} = I_{a2} = I_{a0} = \dfrac{E_a}{Z_1 + Z_2 + Z_0 + 3Z_f}$

$$= \frac{\frac{400}{\sqrt{3}} \angle 0^\circ}{j(4+2+1+6)}$$

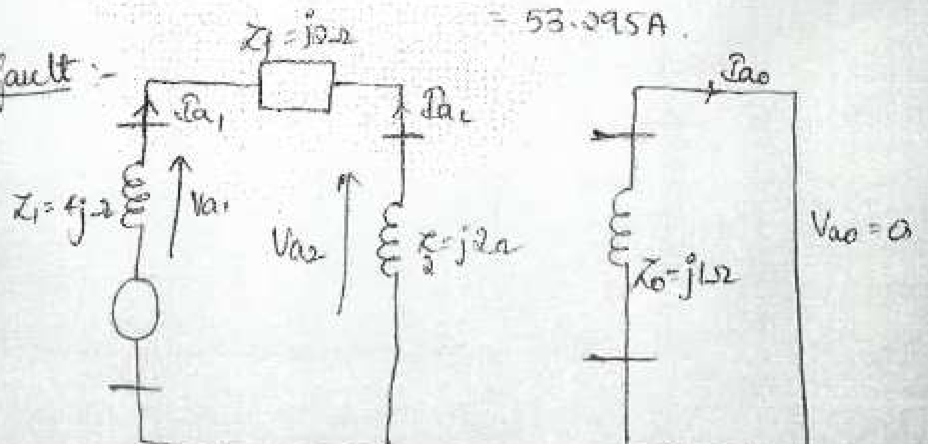
$$= -j17.765A$$

fault current $= I_f = 3|I_{a0}|$

$= 3(17.765)$

$= 53.295A$.

(ii) LL fault :-



Here, $I_{a1} = \dfrac{E_a}{Z_1 + Z_2 + Z_f}$

$$\Rightarrow \frac{400/\sqrt{3}\ \angle 0^\circ}{j(4+2+2)}$$

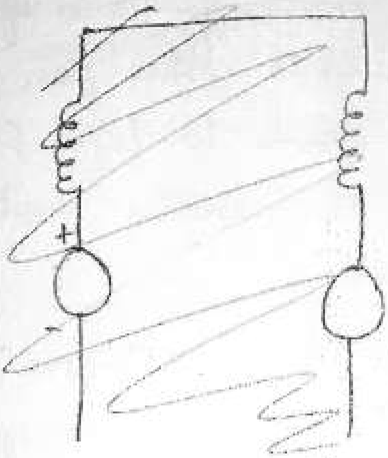
$$= -j28.87\,A$$

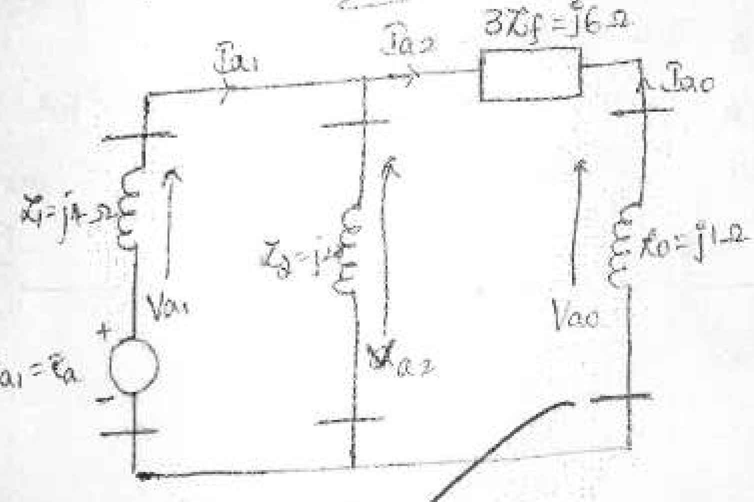
$\therefore$ fault current, $I_f = \sqrt{3}\,|I_{a1}|$

$$= \sqrt{3}\,(28.87)$$

$$= 50A$$

(iii)





Here $I_{a1} = \dfrac{E_a}{Z_1 + \dfrac{Z_2(Z_0+3Z_f)}{Z_2+Z_0+3Z_f}}$

$$= \frac{400/\sqrt{3}\ \angle 0^\circ}{j\left[4 + \dfrac{2(1+6)}{2+1+6}\right]}$$

$$= -j41.57\,A$$

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Sequence reactance of Generator

$$x_1 = 0.2 \times 20/20 \times 11^2/11^2 = 0.2pu.$$

$$X_2 = 0.1 \times 20/20 \times 11^2/11^2 = 0.1pu.$$

$$X_0 = 0.1 \times 20/20 \times 11^2/11^2 = 0.1pu.$$

Sequence reactance of X^{mer} T_1:-

$$X_1 = X_2 = X_0 = X_2 = \frac{(MVA)_{B\ new}}{(MVA)_{B\ old}} \times \frac{(KV)^2_{B\ old}}{(KV)^2_{B\ new}}$$

$$= 0.1 \times \left(\frac{20}{18}\right) \times \left(\frac{11.5^2}{11^2}\right)$$

$$= 0.12pu$$

X_L :- $X_1 = X_2 = X_1 \text{ in } \Omega \times \frac{(MVA)_{B\ new}}{(KV_B)^2}$

$$= 5 \times \frac{20}{(33)^2} = 0.092pu$$

$$X_0 = 10 \times \frac{20}{(33)^2} = 0.184pu.$$

T_2:-

$$X_1 = X_2 = X_0 = X \Rightarrow 0.1 \times \frac{20}{15} \times \frac{(6.9)^2}{(6.6)^2}$$

$$\Rightarrow 0.146pu.$$

Motor:

$$X_1 = 0.2 \times \frac{20}{15} \times \frac{6.9^2}{6.6^2} \Rightarrow 0.27pu.$$

$$X_2 = X_0 = 0.1 \times \frac{20}{15} \times \frac{6.9^2}{6.6^2} \Rightarrow 0.145pu$$

∴ Using current division eq^n

$$I_{a0} = -I_{a1}\left[\frac{Z_2}{Z_2+Z_0+3Z_f}\right]$$

$$= j41.57\left[\frac{2}{2+1+6}\right] \Rightarrow j9.24\,A$$

Hence, $I_f = 3|I_{a0}|$

$= 3(9.24)$

$= 27.72$ A Ans

Q4 A Synchronous motor is receiving 10MW of power at 0.8 pf lag at 6KV. An LG fault takes place at the middle point of the TL as shown in fig. Find the fault current. The rating of the generator motor & x^mer are as under:

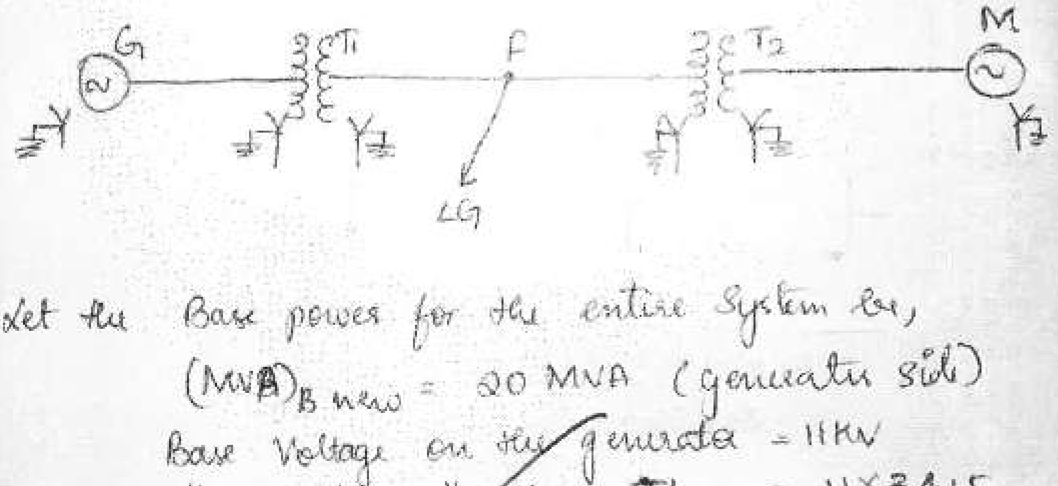
Generator : 20MVA, 11KV, $X_1 = 0.2$pu, $X_2 = 0.1$pu, $X_0 = 0.1$pu

x^mer : T_1 = 18MVA, 11.5Y − 34.5Y KV, X = 0.1pu.

TL : $X_1 = X_2 = 5\Omega$, $X_0 = 10\Omega$.

x^mer T_2 : 15MVA, 6.9Y − 34.5Y KV, x = 0.1pu

Motor : 15MVA, 6.9KV, $X_1 = 0.2$pu, $X_2 = 0.1$pu.



Ans Let the Base power for the entire System be,

$(MVA)_{B\,new}$ = 20 MVA (generator side)

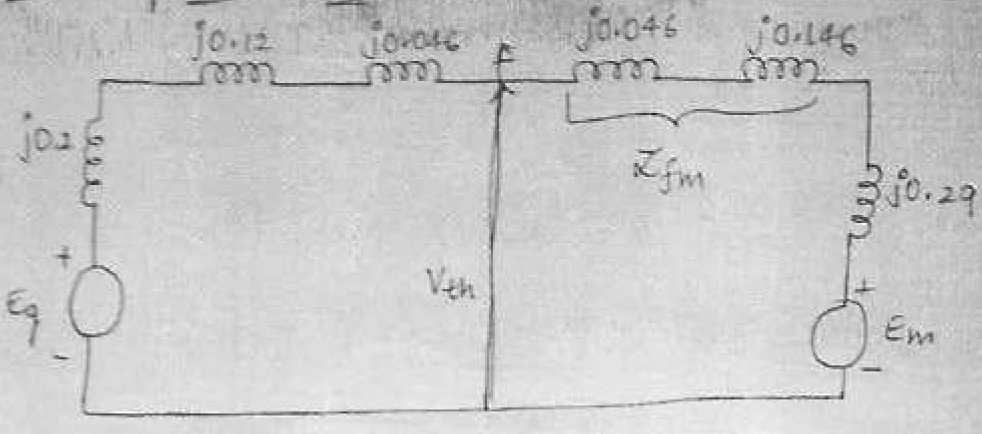
Base Voltage on the generator = 11KV

" " " " TL $= 11 \times \frac{34.5}{11.5}$

$= 33$KV

— " — Motor $= 33 \times \frac{6.9}{34.3} = 6.6$KV

Positive Sequence Network



To find the Voltage at the fault point :- (V_{th})

The current drawn by the motor $I_m = \dfrac{10\times10^6}{\sqrt{3}\times 6\times10^3\times 0.8} \angle -\cos^{-1}0.8$

$= 1202.8 \angle -36.87^\circ A$

The Base current in the motor $(I_m)_B = \dfrac{20\times10^6}{\sqrt{3}\times 6.6\times10^3} = 1749.55A$

$\therefore I_m$ in pu $= \dfrac{I_m}{(I_m)_B} = \dfrac{1202.8}{1749.55} \angle -36.87^\circ$

$= 0.687 \angle -36.87^\circ$ pu

V_m in pu $= \dfrac{6}{6.6} = 0.909 \angle 0^\circ$ pu

Hence the Voltage at the fault point is

$V_{th} = V_m + I_m Z_{fm}$

$= 0.909 + (0.687 \angle -36.87^\circ) \times (0.192 \angle 90^\circ)$

$= 0.994 \angle 6.1^\circ$ pu

To find the Thevenin's impedance Z_{th}

$Z_{th} \Rightarrow j[(0.2 + 0.12 + 0.046) \parallel (0.046 + 0.146 + 0.29)]$

$\Rightarrow j(0.366 \parallel 0.482)$

$\Rightarrow j0.208$ pu

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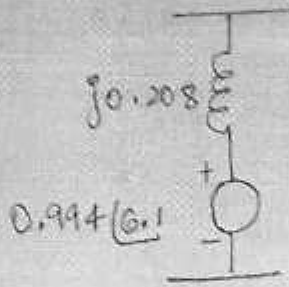
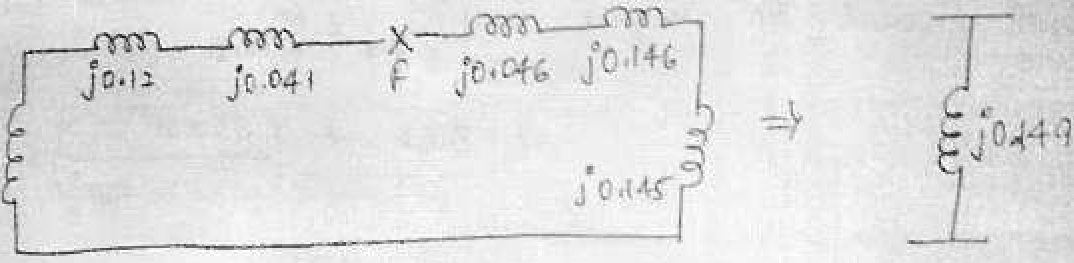
Hence the Equivalent PSN of the System is shown



Negative Sequence Network :-



$Z_{th} = j\,[0.1 + 0.12 + 0.046] \parallel [0.046 + 0.146 + 0.145]$

$= j\,[(0.266) \parallel (0.337)]$

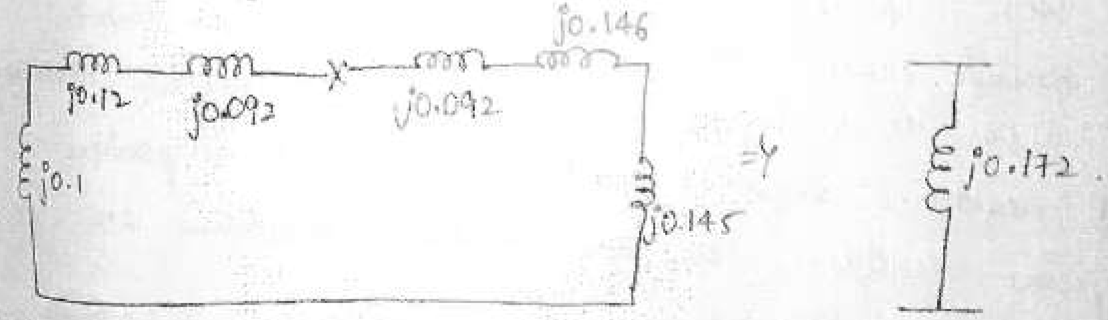
$= j\,(0.149\,pu)$

Zero Sequence Network (ZSN)

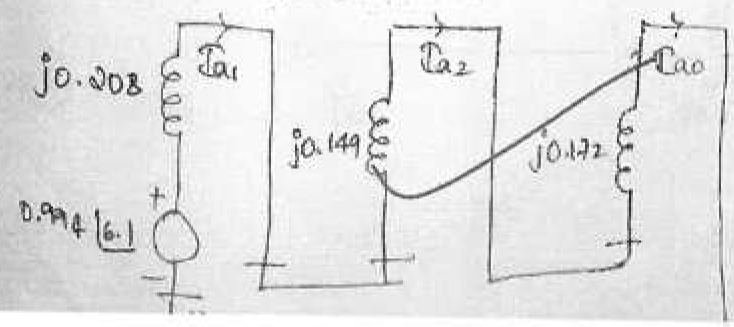
$Z_{0th} = j\,[0.1 + 0.12 + 0.092) \parallel (0.092 + 0.146 + 0.145)]$

$= j\,(0.312) \parallel (0.383)$

$= j0.172\,pu.$



InterConnection of Sequence networks



Here, $I_{a1} = I_{a2} = I_{a0} = \dfrac{0.994\angle 6.1^\circ}{j(0.208 + 0.149 + 0.172)}$

$= 1.88\angle -83.9^\circ \ pu$

Hence fault current $|I_f|_{pu} = 3|I_{a0}|$

$= 3\times(1.88)$

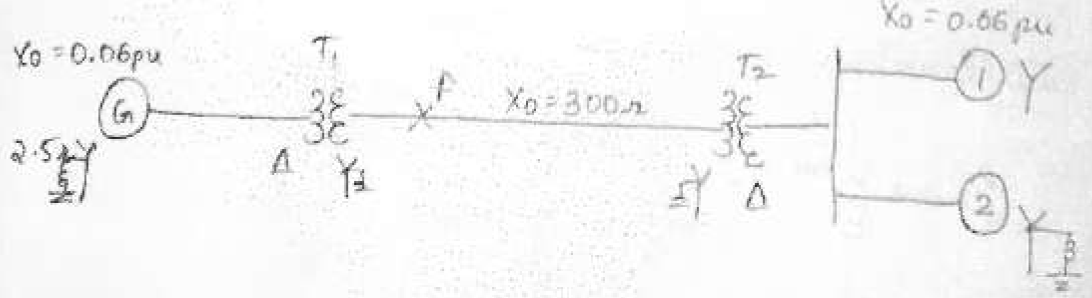
$= 5.64\ pu.$

fault current in ampere is $|I_f|_{pu} \times (I_{TL})_B$

$\Rightarrow 5.64 \times \left(\dfrac{20\times10^6}{\sqrt{3}\times 33\times10^3}\right)$

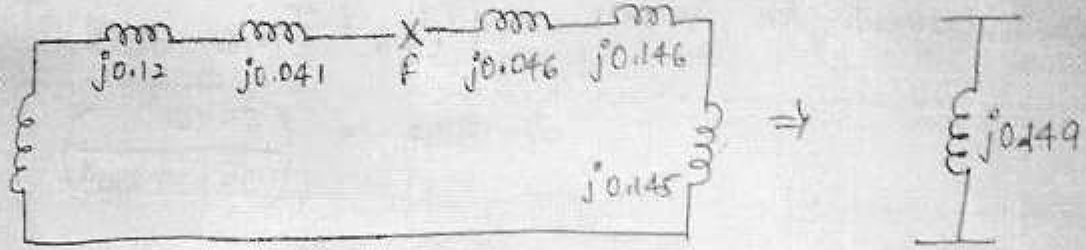
$\Rightarrow 1973.49A$

(3) A 25MVA, 11KV, 3-ϕ generator has a subtransient reactance of 20%. The generator supplies 2 motors over a transmission line with X^{mer} at both ends as shown the. The motors have rated input of 15 & 7.5MVA, Both 10KV with 25% subtransient reactance. The 3-ϕ X^{mer} are Both rated 30MVA, 10.8/121kV, connection Δ-Y with leakage reactance of 10% each. The series reactance of line is 100Ω. Calculate the fault current when a single line to ground fault occurs at F. The motors are loaded to also 15 & 7.5MW at 10KV & 0.8pf. loading, Assume that negative sequence reactance is equal to positive sequence reactance. The zero sequence reactance are shown. Omit resistance.



Hence the Equivalent PSN of the System is shown

Negative Sequence Network :-



$Z_{2th} = j[0.1 + 0.12 + 0.046] \parallel [0.046 + 0.146 + 0.145]$

$= j[(0.266) \parallel (0.337)]$

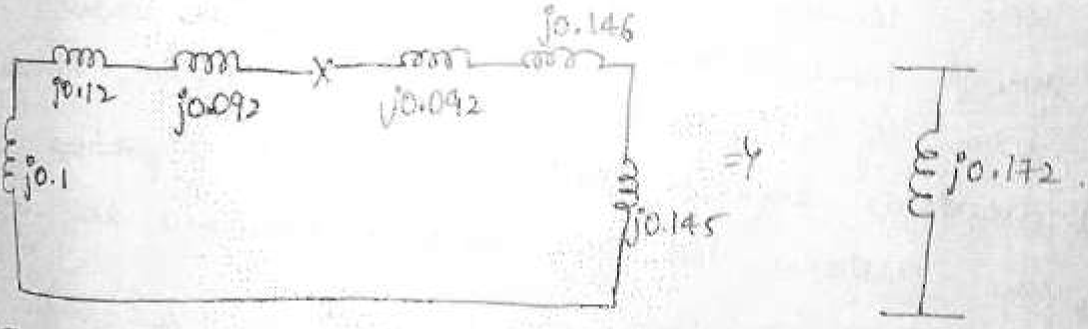
$= j(0.149pu)$.

Zero Sequence Network (ZSN)

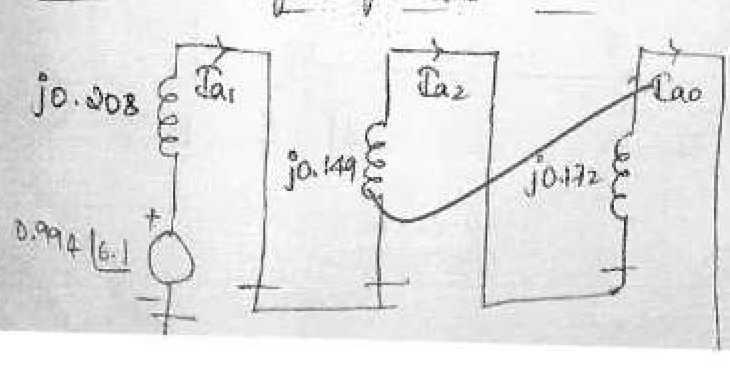
$Z_{0th} = j[0.1 + 0.12 + 0.092) \parallel (0.092 + 0.146 + 0.145)]$

$= j(10.312) \parallel (0.383)$

$= j0.172pu$.



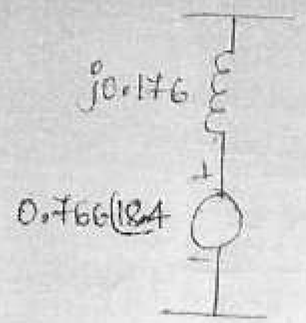
Inter Connection of Sequence networks



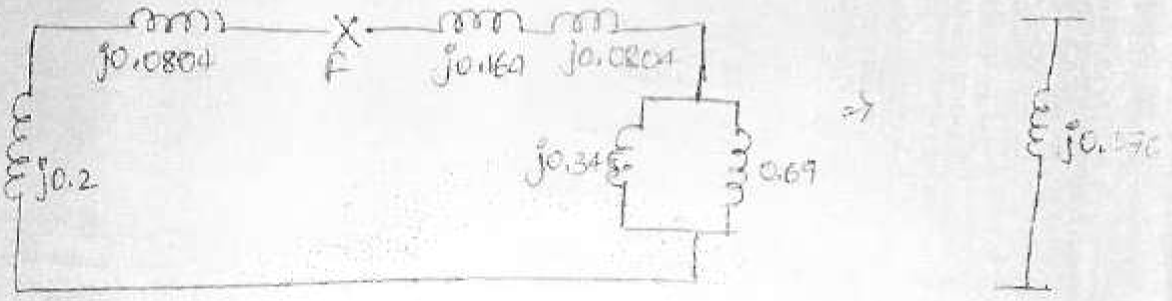
To find Z_{th}

$$Z_{th} = j[(0.2 + 0.0804) \parallel (0.164 + 0.0804 + 0.23)]$$
$$= j0.176\ pu$$

Hence the Equivalent PSN shown

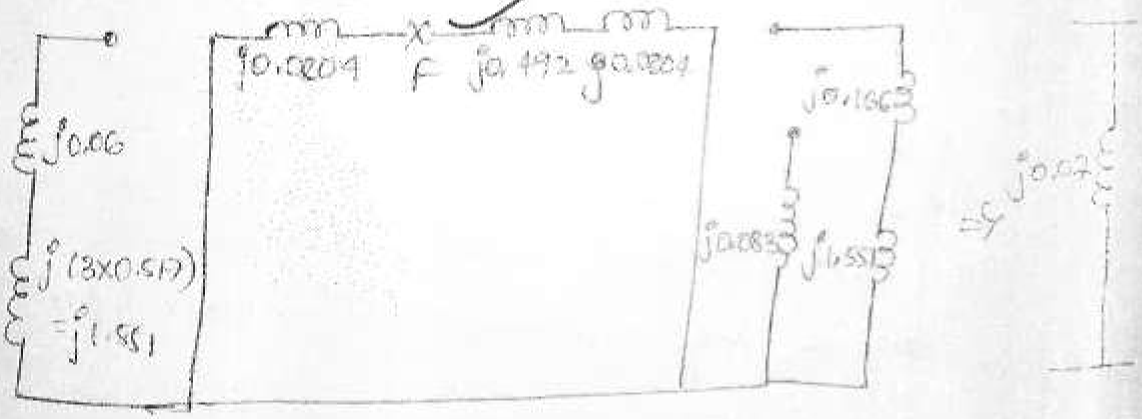


Negative Sequence Network



$$Z_{TH} = j[(0.2 + 0.0804) \parallel (0.164 + 0.0804 + 0.23)]$$
$$= j0.176\ pu$$

Zero Sequence Network (ZSN)

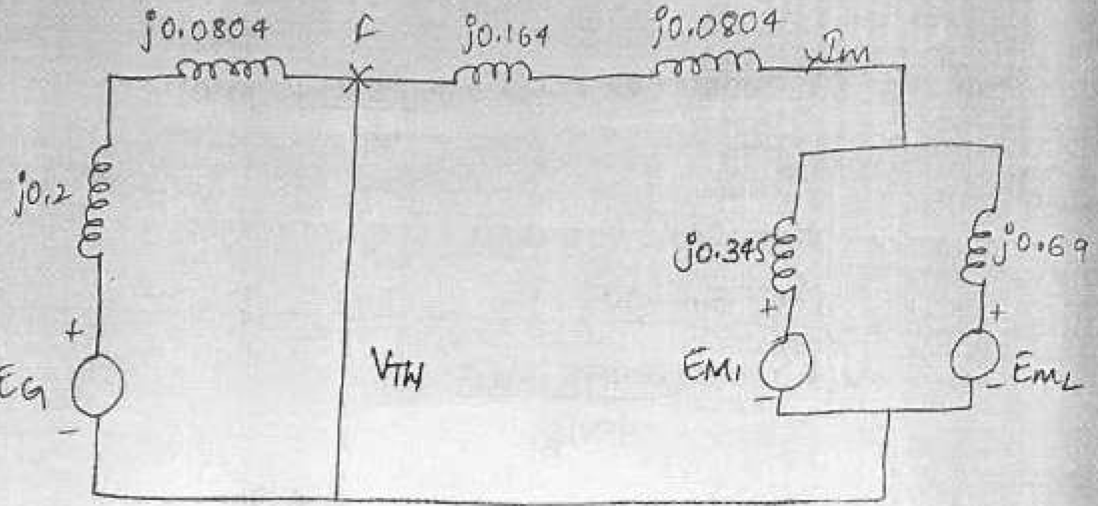


$$Z_{0TH} = j[0.0804 \parallel (0.492 + 0.0804)]$$
$$= j0.07\ pu$$

M_2 =:
$$X_1 = X_2 = 0.25 \times \frac{25}{7.5} \times \frac{(10)^2}{(11)^2} = 0.69\,pu$$

$$X_0 = 0.06 \times \frac{25}{15} \times \frac{10^2}{11^2} = 0.166\,pu$$

Positive Sequence Network :-



Let us find V_{TH} first

$$I_m = \frac{(15+7.5)\times 10^6 \angle \cos^{-1} 0.8}{\sqrt{3} \times 10 \times 10^3 \times 0.8}$$
$$= 1623.8 \angle 36.87^\circ A$$

The Base current in the motor ckt $= (I_m)_B = \dfrac{25 \times 10^2}{\sqrt{3} \times 11 \times 10^3}$
$$= 1312.2A$$

$$\therefore I_m \text{ in P.u} = \frac{I_m}{(I_m)_B} = \frac{1623.8 \angle 36.87}{1312.2} = 1.24 \angle 36.87^\circ$$

$$V_m \text{ in pu} = \frac{10}{11} = 0.909\,pu.$$

Hence the voltage at the fault point is

$$V_{TH} = V_m + I_m \cdot Z_m$$
$$= 0.909 + 1.24 \angle 36.87^\circ (0.164 + 0.0804) \angle 90^\circ$$
$$= 0.766 \angle 18.4^\circ\,pu$$

for 3 ф fault.

Here, $I_{a1} = \frac{E_a}{Z_1} = \frac{11\times10^3/\sqrt{3}}{X_1} \longrightarrow (1)$

In this case $|I_f| = |I_{a1}| = 2000A$

Sub this in (1), we get

$$2000 = \frac{11\times10^3/\sqrt{3}}{X_1}$$

$$X_1 = 3.175\,\Omega$$

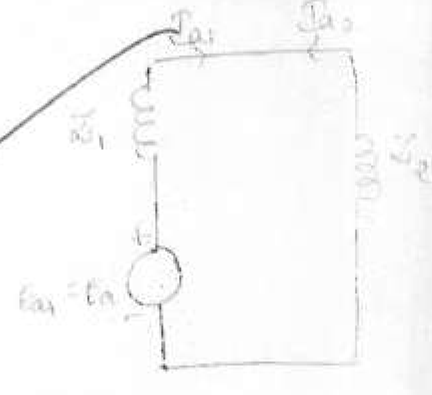
Also X_1 in pu $= X_1$ in $\Omega \times \frac{(MVA)_B}{(KV)_B^2}$

$(MVA)_B = 50MVA \qquad (KV)_B = 11KV$

X_1 in pu $= 3.175 \times \frac{50}{11^2} = 1.31$ pu.

line to line fault:-

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} = \frac{11\times10^3/\sqrt{3}}{(X_1+X_2)} \longrightarrow (2)$$



Given $|I_f| = 1800A$

$\therefore I_{a1} = \frac{1800}{\sqrt{3}} = 1039.23A$

Sub the value of I_{a1} in (2), we get

$$1039.23 = \frac{11\times10^3/\sqrt{3}}{X_1 + X_2}$$

$$X_1 + X_2 = \frac{11\times10^3/\sqrt{3}}{1039.23} = 6.11\,\Omega$$

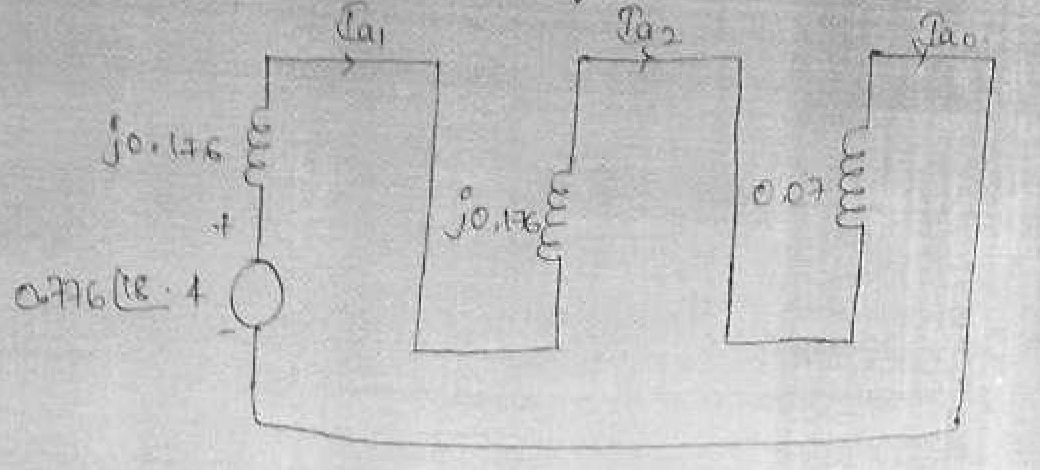
but $X_1 = 3.175\,\Omega$.

Hence

$X_2 = 6.11 - 3.175 = 2.935\,\Omega$.

X_2 in pu $= 2.935 \times \frac{50}{11^2} = 1.21$ pu

Interconnection of Sequence Network



Here, $I_{a1} = I_{a2} = I_{a0} = \dfrac{0.766\angle 18.4}{j(0.176 + 0.176 + 0.07)}$

$= 1.815 \angle -71.6^\circ$ pu

$|I_f| = 3|I_{a0}|$

$= 3 \times 1.815$

$= 5.445$ pu.

I_f in ampere is given as $= |I_f| = 5.445 \times \left(\dfrac{25 \times 10^6}{\sqrt{3} \times 123.2 \times 10^3}\right)$

$|I_f| = 637.92$ A

(4) A 3-ϕ, 50MVA, 11kV, Y-connected neutral solidly grounded generator operating on an no load at rated voltage gave the following sustained fault current for the faults specified.

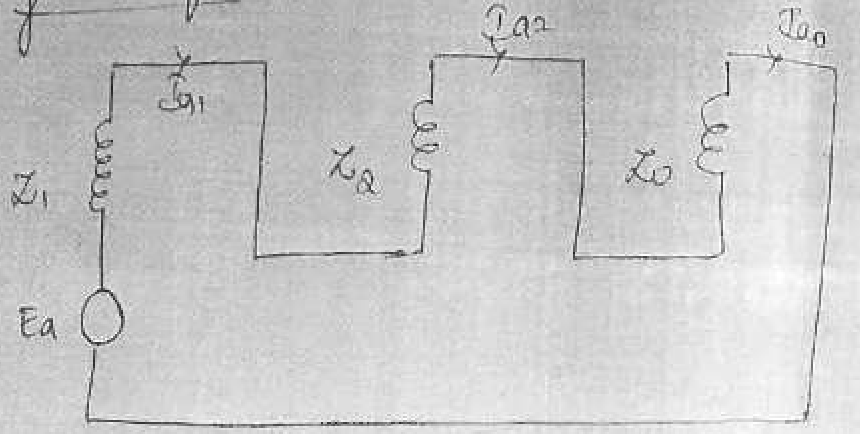
3-ϕ, fault - 2000A

line to line fault - 1800A

line to ground fault - 2200A

Determine the 3 sequence reactance in ohms & p.u.

Line to ground fault :-



Here , $I_{a1} = I_{a2} = I_{a0} = \dfrac{E_a}{Z_1 + Z_2 + Z_0}$

$= \dfrac{11 \times 10^3 / \sqrt{3}}{X_1 + X_2 + X_0}$

$\therefore \ |I_F| = 3\,|I_{a0}|$

Given $|I_F| = 2200$

Hence $I_{a0} = \dfrac{2200}{3}$

$= 733.33A$

Sub this value in (3) we get

$733.33 = \dfrac{11 \times 10^3 / \sqrt{3}}{X_1 + X_2 + X_0}$

$X_1 + X_2 + X_0 = 8.66\,\Omega$

$X_1 = 3.175\,\Omega \qquad X_2 = 2.935\,\Omega$

Hence $X_0 = 8.66 - 3.175 - 2.935 \Rightarrow 2.55\,\Omega$

X_0 in pu $= 2.55 \times \dfrac{50}{(11)^2} = 1.0545$ pu.

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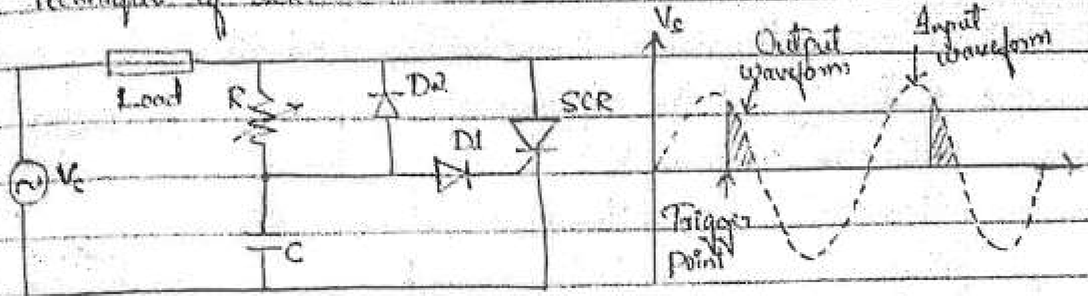
Assignment 01

1. With a neat circuit diagram and waveforms explain RC Triggering Technique of SCR.



* The limitation of resistance firing Circuit Can be overcome by the RC Triggering Circuit which provides the firing angle Control from 0 to 180 degrees. By changing the phase and amplitude of the gate current, a large variation of firing angle is obtained using this circuit.

* The above figure shows the RC Triggering Circuit Consisting of two diodes with an RC network connected to turn the SCR.

* By varying the variable resistance triggering or firing angle is Controlled in a full positive half cycle of the input signal.

* During the negative half cycle of the input signal, Capacitor charges with lower plate positive through diode D2 up To the maximum supply voltage Vmax. This voltage remains at -Vmax across the Capacitor till supply voltage attains zero crossing.

* During the positive half cycle of the input, the SCR becomes forward biased and the Capacitor starts charging through variable resistance to the triggering voltage value of the SCR.

* When the Capacitor charging voltage is equal to the gate trigger voltage, SCR is Turned ON and the Capacitor holds a small voltage. Therefore the capacitor voltage is helpful for triggering the SCR even after 90 degrees of the input waveform.

* In this diode D1 prevents the negative voltage between the gate and Cathode during the negative half cycle of the

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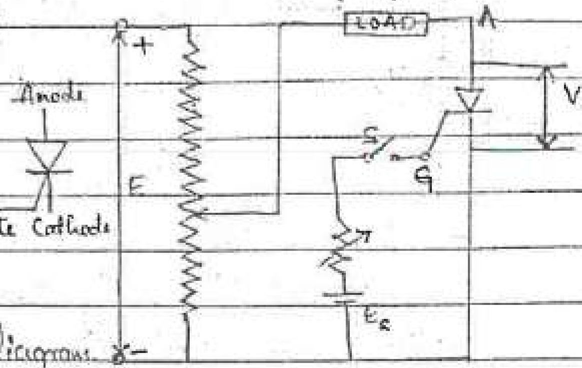
input through diode D2.

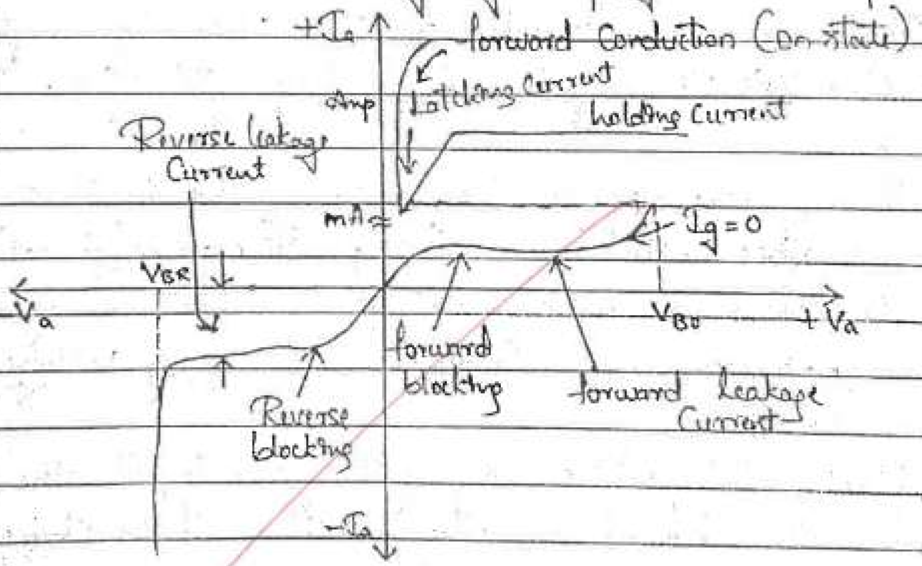
2. Explain thyristor characteristics and its mode of operation

→ * On giving the supply we get the required V-I characteristics of a thyristor shown in the fig. for anode to cathode voltage V_a & anode current I_a as we can see from the circuit diagram.



Circuit diagram

* Initially for the reverse blocking mode of the thyristors, the cathode is made positive with respect to anode by supplying voltage E and the gate to cathode supply voltage E_s is detached initially by keeping switch 'S' open.



* When the anode to cathode forward voltage is increased with gate circuit open, the reverse junction J_2 will have an avalanche breakdown at forward break over voltage V_{BO} leading to thyristor turn on.
* Once the thyristor is turned on we can see from the diagram for characteristics of thyristor, that the point N_o at once shifts toward N and then anywhere b/w N & K.

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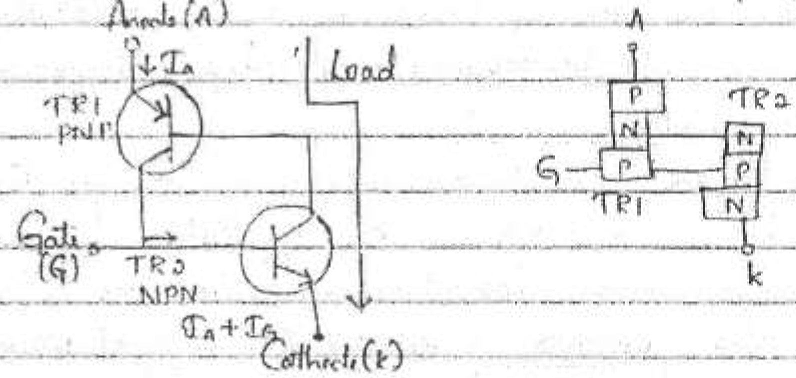
* Here MK represents the forward conduction mode of operation, the thyristor conducts maximum current with minimum voltage drop, this is known as the forward conduction forward or the turn on mode of the thyristor.

3. Using two transistor model, explain how a small gate current can turn-on the SCR when blocking forward voltage.

→ 

* The two transistor equivalent circuit shows that the collector current of the NPN transistor TR_2 feeds directly into the base of the PNP transistor TR_1, while the collector current of TR_1 feeds into the base of TR_2.
* These two inter-connected transistors rely upon each other for conduction as each transistor gets its base-emitter current. So until one of the transistor gets ~~its to~~ is given some base current nothing can happen even if an anode to cathode voltage is present.
* If the anode terminal is made positive with respect to the cathode, the two outer P-N junctions are now forward biased but the centre N-P junction is reverse biased. ∴ forward current is also blocked. If a positive current is injected into the base of the NPN transistor TR_2, the resulting collector current flows in the base of transistor TR_1. This in turn causes a collector current to flow in the PNP transistor, TR_1 which increases the base current of TR_2 and so on.

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* Thus a thyristor blocks Current in both directions of an AC supply in its OFF state and can be turned ON and made to act like a normal rectifying diode by the application of a positive current to the base of transistor TR2 which for a silicon controlled rectifier is called the "Gate" Terminal.

4. Discuss the need of protection against di/dt & dv/dt. Explain how it is achieved with suitable diagrams.

→ di/dt protection of SCR.

* The anode current starts flowing through the SCR when it is turned ON by the application of gate signal. This anode current takes some finite time to spread across the junctions of an SCR. For a good working of SCR, this current must spread uniformly over the surface of the junction.
* If the rate of rise of anode current (di/dt) is high results a non-uniform spreading of current over the junction.
* Due to the high current density, this further leads to form local hot spots near the gate - cathode junction.
* This effect may damage the SCR due to overheating. Hence, during turn ON process of SCR, the di/dt must be kept below the specified limits.
* Typical range between 20-500 ampere per microseconds.

dv/dt protection of SCR.

* When the SCR is forward biased, junctions J1 and J3 forward biased and junction J2 is reverse biased. This reverse biased junction J2 exhibits the characteristics of a capacitor. Therefore, if the rate of forward voltage applied is very high across the SCR, charging current flows through the junction J2 is high enough to turn ON the SCR even without any gate signal.

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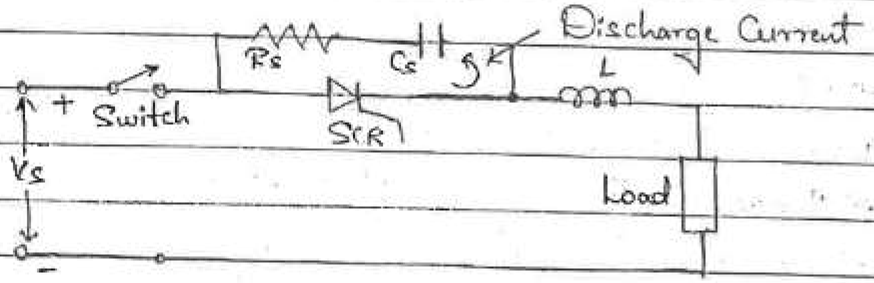
* Hence, the rate of rise of anode to Cathode voltage, dv/dt must be in specified limit to protect the SCR against false Triggering. This can be achieved by using RC snubber network across the SCR.



* The protection against high voltage reverse recovery transient -s and dv/dt is achieved by using an RC snubber circuit.
* This snubber circuit consists of a series combination of Capacitor & resistor which is connected across the SCR
* This also consist an inductance in series with the SCR to prevent the high di/dt. The resistance value is of few hundr -ed ohms

5. What are firing circuits explain various types of firing circuits.

→ The application of gate voltage is known as firing.

Types of SCR firing:

Generally, there are two types of firing:

* Zero voltage cross over firing :- Zero-crossing control mode (also called fast cycling, integral cycle, or burst firing) operates by turning the SCR's on only when the instantaneous value of the sinusoidal voltage is zero
* Phase angle control methods :- The phase angle is varied, i.e the application of gate pulses is delayed by a certain time and the conduction is controlled.

Types of firing circuit:

• R-firing circuit • RC firing circuit

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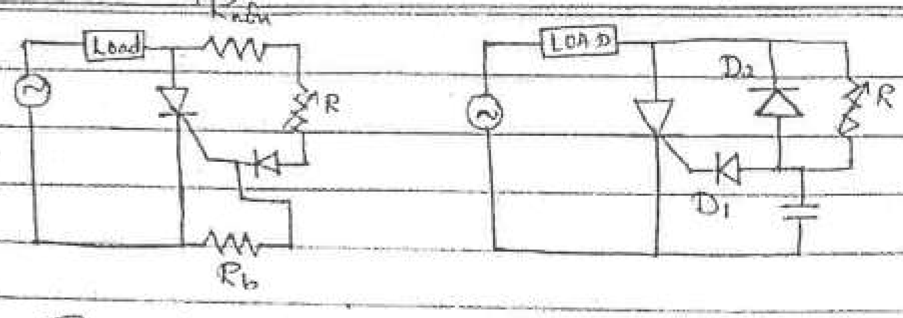
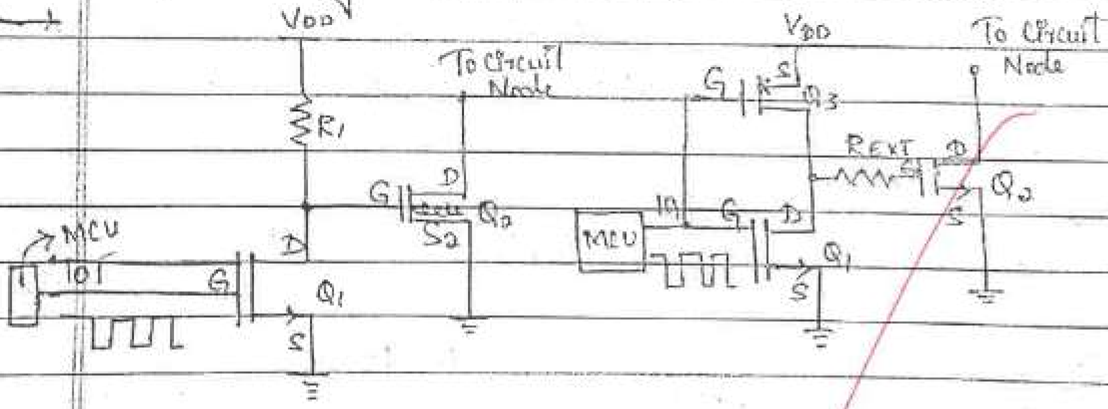


* The number of degrees from the beginning of the cycle when SCR is switched on is firing angle.
* Phase shifting gate control: It causes 0 to 180° delay of conduction. The phase angle of the gate voltage is changed with respect to the anode-cathode voltage.

6. Discuss the needs and methods for providing isolation of gate/base circuits from power circuit with necessary circuit diagram.



* The structure of IGBT/power MOSFET is such that the gate forms a nonlinear capacitor. Charging the gate capacitor turns the power device on and allows current flow between its ~~eb~~ drain and source terminals, while discharging it turns the device off and a large voltage may then be blocked across the drain and source terminals.
* For operating an IGBT / power ~~Mosfet~~ MOSFET as a switch, a voltage sufficiently larger than V_{TH} should be applied b/w the gate and source/emitter terminals.
* As in figure, when IO sends out a low signal, $V_{GSQ1} < V_{THQ1}$ and thus MOSFET Q_1 remains off. As a result,

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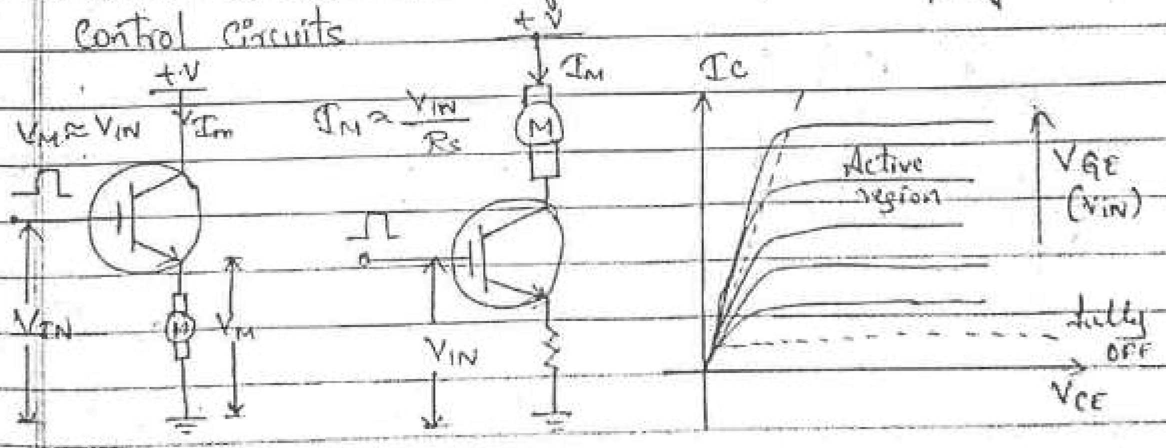
a positive vtg is applied at the gate of power MOSFET Q_2.
* The gate capacitor of Q_2 (C_{GQ_2}) charges through pull-up resistor R_1 and the gate voltage is pulled to the rail vtg of V_{DD}.
* Given $V_{DD} > V_{THQ_2}$, Q_2 turns on and can conduct. When IC outputs high, Q_1 turns on and C_{GQ_2} discharges through Q_1, $V_{DSQ_1} \sim 0V$ such that $V_{GSQ_2} < V_{THQ_2}$ and hence, Q_2 turns off. One issue with this setup is of power dissipation in R_1 during on state of Q_1.
* To overcome this, P MOSFET Q_3 can be used as a pull up to operate in a complementary fashion with Q_1.
* This gate driver IC will almost always have additional internal circuits for greater functionality, but it primarily works as a power amplifier and a level shifter.

7. What is an IGBT? Draw its switching characteristics. What are its advantages over BJT and MOSFET.

→ Insulated Gate Bipolar Transistor (IGBT) is a power switching ~~transistor which~~ combines the advantages of MOSFET's and BJT's for use in power supply and motor control circuits.



* Since the IGBT is a voltage controlled device, it only requires a small voltage on the gate to maintain conduction through the device unlike BJT's which require

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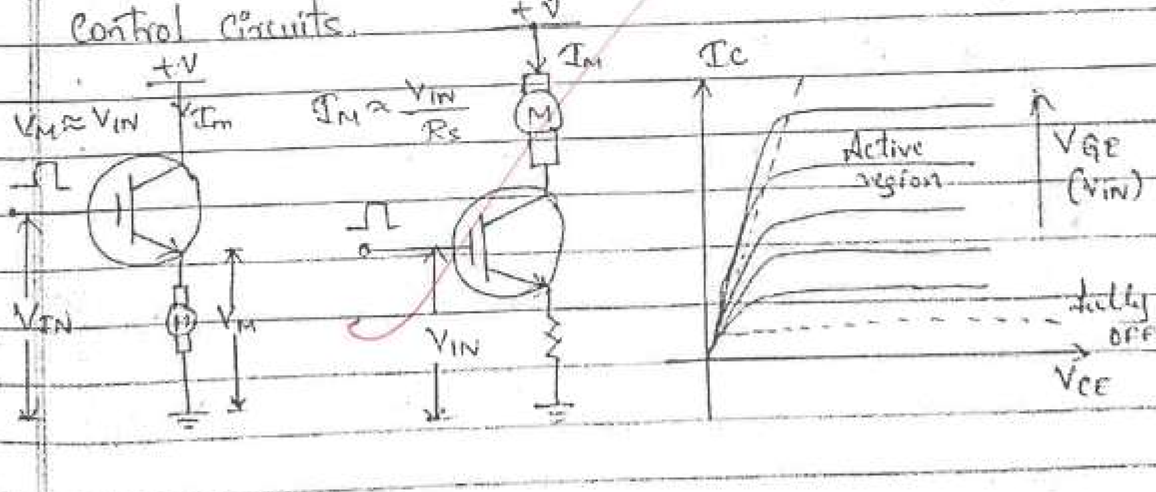


* Since the IGBT is a voltage controlled device, it only requires a small voltage on the Gate to maintain conduction through the device unlike BJT's which require

The symbols used in the above i/p & o/p waveforms are briefly explained below:

1) $t_{d(on)}$: Turn on delay time

The time from when the gate-source voltage rises over 10% of V_{GS} until the drain-source voltage reaches 90% of V_{DS}

2) t_r: Rise time

The time taken for the drain-source voltage to fall from 90% to 10% of V_{DS}

3) t_{on}: Turn-on time.

The turn-on time is equal to $t_{d(on)} + t_r$.

4) $t_{d(off)}$: Turn-off delay-time.

The time from when the gate-source voltage drops below drain-source voltage reaches 10% of V_{DS}.

5) t_f: fall time

The time taken for the drain-source voltage to rise from 10% to 90% of V_{DS}

6) t_{off}: Turn-off time

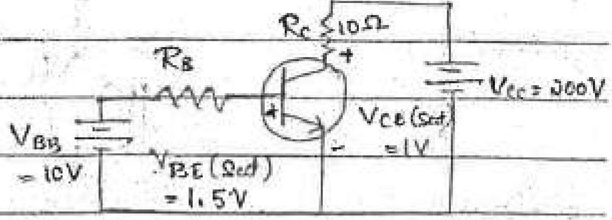
The turn-off time is equal to $t_{d(off)} + t_f$.

9. A Transistor switch as shown in fig has β in the range of 8-40. Calculate

i) The value of R_B that results in saturation with an overdrive factor of 5.

ii) The forced β_f and

iii) The power loss in the transistor.



Sol^n:- $\beta = 8$ to 40, $ODF = 5$, $R_C = 10\,\Omega$, $V_{CE(sat)} = 1V$, $V_{BE(sat)} = 1.5V$, $V_{CC} = 200V$, $V_{BB} = 10V$

i) To Obtain value of R_B

from collector - emitter loop

$$I_{C(sat)} = \frac{V_{CC} - V_{CE(sat)}}{R_C} = \frac{200-1}{10} = 19.9A$$

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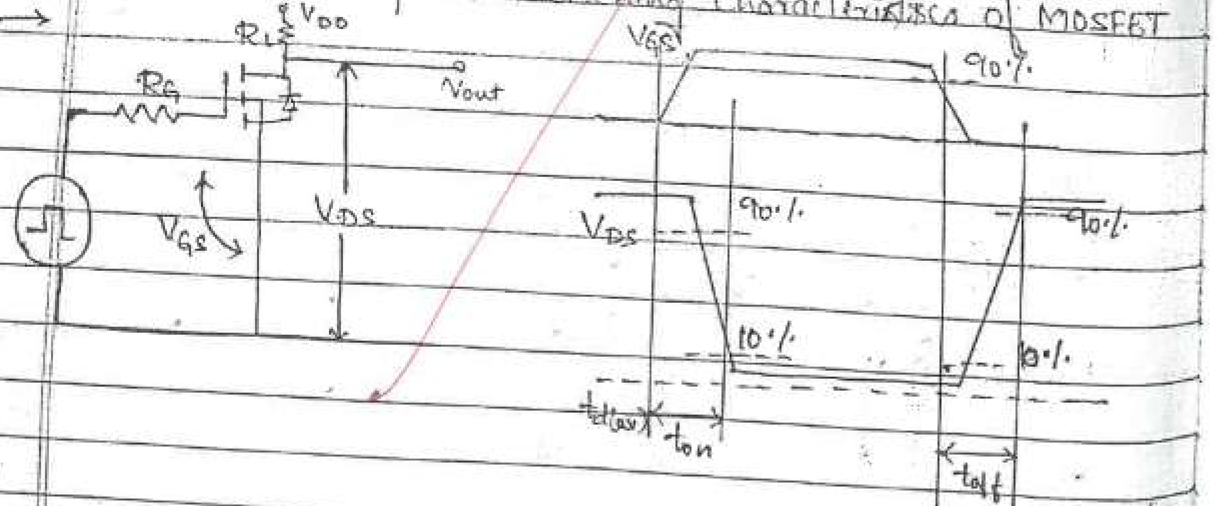
that the base current is continuously supplied in a sufficient enough quantity to maintain saturation.
* The principle of operation and Gate drive circuits for the insulated gate bipolar transistor are very similar to that of the N-channel power MOSFET. The basic difference is that the resistance offered by the main conducting channel when current flows through the device in its "ON" state is very much smaller in the IGBT.
* The main advantage of using the IGBT over other types of Transistor devices are its high voltage capability low ON resistance, ease of drive, relatively fast switching speeds & combined with zero gate drive current makes it a good choice for moderate speed, high voltage applications such as in pulse-width modulated, (PWM) variable speed control, switch-mode power supplies or solar powered DC-AC inverter and frequency converter applications operating in the hundreds of kilohertz range.

8. Draw and explain switching characteristics of MOSFET



* Since power MOSFET's are majority-carrier devices, they are faster and capable of switching at higher frequencies than bipolar transistors.

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$$\therefore I_o(av) = \frac{\delta(V_S - V_{CE(sat)})}{R} = 0.7 \times \frac{(200-2)}{10} = 13.86\,A$$

ii) Conduction power loss

$$P_{loss} = \frac{1}{T}\int_0^{T_{on}} (I_C \times V_{CE})\,dt = \frac{T_{on}}{T}(I_C \times V_{CE})$$

$$= \delta \times I_o(av) \times V_{CE} \qquad \because \frac{T_{on}}{T} = \delta \Rightarrow I_C = I_o(av)$$

$$= 0.7 \times 13.86 \times 2 = 19.404\,W$$

iii) Switching power loss

$$P_{on}\ loss = \frac{1}{T}\int_0^{t_{on}} (I_C \times V_{CE})\,dt = \frac{t_{on}}{T} \times I_C \times V_{CE}$$

$$= t_{on} \times f_s \times I_o(av) \times V_{CE} \quad \text{Since } I_C = I_o(av)\ \&\ f_s = 1/T$$

$$= 3\times10^{-6} \times 1\times10^{3} \times 13.86 \times 2 = 0.08316\,W \text{ or } 83.16\,mW$$

$$P_{off}\ loss = \frac{1}{T}\int_0^{t_{off}} \cancel{(I_C \times V_{CE})\,dt} = \cancel{\frac{t_{off}}{T} \times I_C \times V_{CE}}$$

$$= t_{off} \times f_s \times I_o(av) \times V_{CE}$$

$$= 1.2\times10^{-6} \times 1\times10^{3} \times 13.86 \times 2$$

$$= 0.03326 \text{ or } 33.26\,mW$$

11. A Capacitor used in the UJT oscillator circuit is charged by a constant current source. The value of the capacitor is 0.5 micro farad and that of the constant current is 1mA. The sawtooth voltage of oscillator is found to have a crest value of 8.5 volts and valley level of 2.5V. Calculate the frequency of the oscillator.

Sol^n:- $C = 0.5\,\mu F$, $i_c = 1mA$ Const Current Charging

$V_P = 8.5V$, $V_V = 2.5V$

The waveform of capacitor is sawtooth. Hence discharge period can be neglected. The voltage across capacitor given as,

$$V_P = \frac{1}{C}\int_0^T i_c\,dt + V_V$$

$$8.5 = \frac{1}{0.5\times10^{-6}}\int_0^T 1\times10^{-3}\,dt + 2.5$$

$I_{B(sat)} = \frac{I_{C(sat)}}{\beta_{min}} = \frac{19.9}{8} = 2.4875\ A$

$ODF = \frac{I_B}{I_{B(sat)}}$

$5 = \frac{I_B}{2.4875} \Rightarrow I_B = 12.4375\ A$

From base-emitter loop,

$V_B = I_B R_B + V_{BE}$

$10 = 12.4375 * R_B + 1.5 \Rightarrow R_B = 0.6834\ \Omega$

ii) To obtain forced β_f

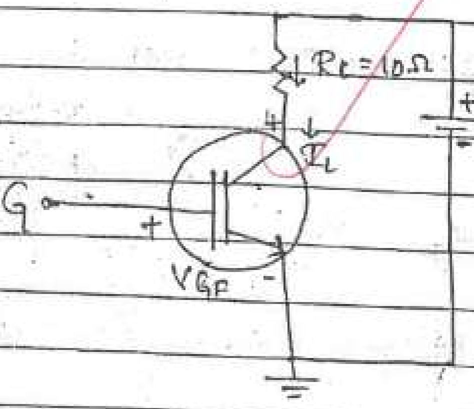
$\beta_{forced} = \frac{I_{C(sat)}}{I_B} = \frac{19.9}{12.4375} = 1.6$

iii) To obtain power loss in the transistor

$P_T = V_{BE} I_B + V_{CE} I_C = 1.5 \times 12.4375 + 1 \times 19.9$

$= 38.55 W$

10. A IGBT shown in fig has the following data: Ton = 3 micro sec, Toff = 1.2 micro sec, duty cycle D = 0.7, $V_{ce(sat)} = 2V$, f = 1kHz. Determine i) Average load current
ii) Conduction power loss
iii) Switching power loss during turn on and turn off



Solⁿ:- $T_{on} = 3\mu Sec$, $T_{off} = 1.2\mu s$, $\delta = 0.7$, $V_{CE(sat)} = 2V$, $f_s = 1kHz$

i) Average load Current

Here, $V_{o(AV)} = \delta(V_c - V_{CE(sat)})$ Since Vtg & load currents are square waves

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$$6 = \frac{1\times10^{-3}}{0.5\times10^{-6}}\int_0^T dt$$

$$\int_0^T dt = 3\times10^{-3}$$

$$[t]_0^T = 3\times10^{-3}$$

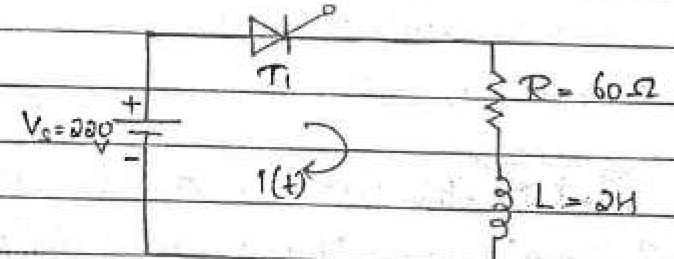
$$T = 3\times10^{-3}$$

frequency, $f = \frac{1}{T} = \frac{1}{3\times10^{-3}}$

$f = \underline{333.33}\ Hz$

12. The thyristor is gated with a pulse width of 40 μSec. The latching current of thyristor is 36 milliampere. For a load of 60 ohms and 2H, will be thyristor get turned ON? If not, how it can be overcome for the given load? Find its value.

Sol^n:



Pulse width = 40 μSec, $I_L = 36mA$, $R = 60\Omega$, $L = 2H$

Assume $V_s = 220V$

Step 1:- To check thyristor current at the end of trigger pulse current through RL circuit is given by

$$i(t) = \frac{V_s}{R}\left(1 - e^{-tR/L}\right)$$

$$= \frac{220}{60}\left(1 - e^{-\frac{40\times10^{-6}\times60}{2}}\right)$$

$$= 4.397\ mA$$

Since this current is less than latching current of 36mA, SCR will not turn-on

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Step 2: Now let us check the time at which SCR Current rises to 36mA.

$$i(t) = \frac{220}{60}\left(1 - e^{-\frac{t \times 60}{2}}\right)$$

$$i(t) = I_L = 36\ mA.$$

$$36 \times 10^{-3} = \frac{220}{60}\left(1 - e^{-\frac{t \times 60}{2}}\right)$$

$$t = 0.329\ msec.$$

Thus, if the triggering pulse is 0.329msec long, then SCR Current will rise to 36mA (latching Current) & it will remain in ON Condition.

1AM17ME068. 2019-20

Prithvi Ponnanna K.S

VI Sem

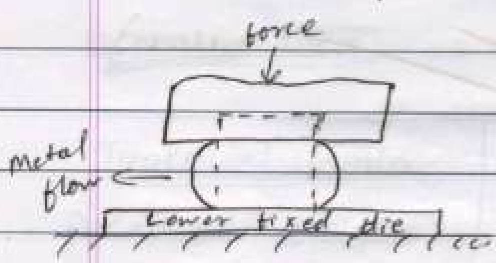
Assignment - I (Metal forming) 17ME653.

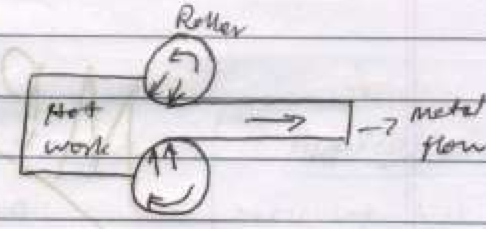
1) Explain classifications of manufacturing process?

Ans. Manufacturing Process can be classified in two different ways as follows:

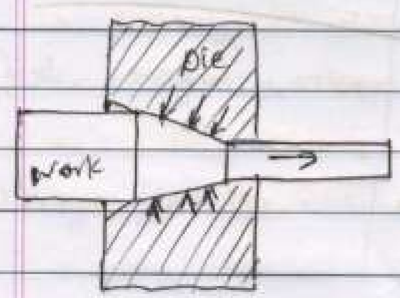
1) Based on the nature of force applied.

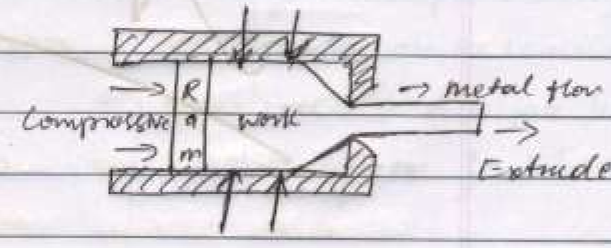
(a) Direct compression Type process.



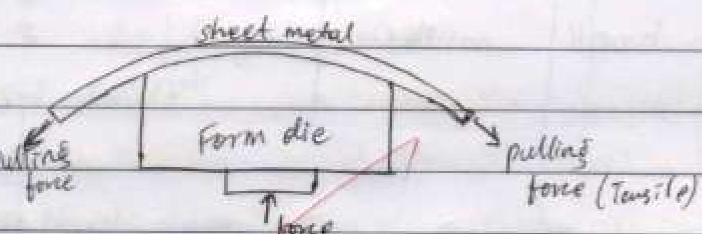


(b) Indirect Compression type.

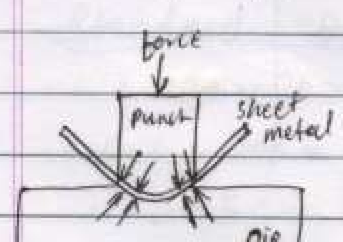




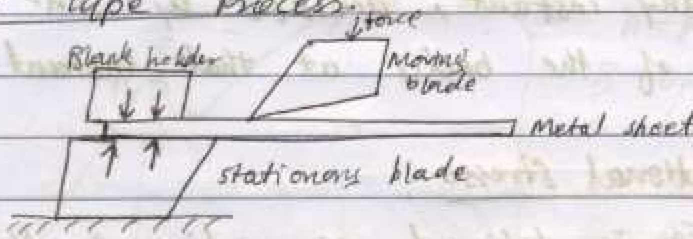
c) Tension Type Process



(c) Bending Type process



c) Shearing Type Process.



(2) Based on Working Temperature

(a) Hot working process.

If the working temp of the workpiece metal is above its recrystallization temp, but below melting point, ~~the~~

(b) Cold Working Process.

If the working temp of the workpiece metal is below its recrystallization temp.

(2). (a) Define Wrought products & write its charact

~~Ans~~ Wrought products refers to those products such as rods, bars, tubes, plates, and wire that are produced by hot or cold working process.

characteristic are:

- Products are formed by deforming the metal in its plastic state.
- Wrought products are ductile in nature.
- Easily identified by their surface finish & sharp corners. Free from, blow holes, porosity.
- Have better mechanical properties & directional flow.
- Exhibits properties in the direction of metal flow.

(b) ~~Be~~Concepts of True stress, conventional stress & Strain.

Ans

True Stress

It is defined as the load acting on the body at any instant, divided by the cross sectional area of the body at that instant.

Conventional Stress:

It is defined as ratio of the force acting on a body to its original cross-sectional area.

$$\boxed{\sigma = \frac{F}{A_0}}$$

Strain:

It is defined as the ratio of change in length to the original length of the body.

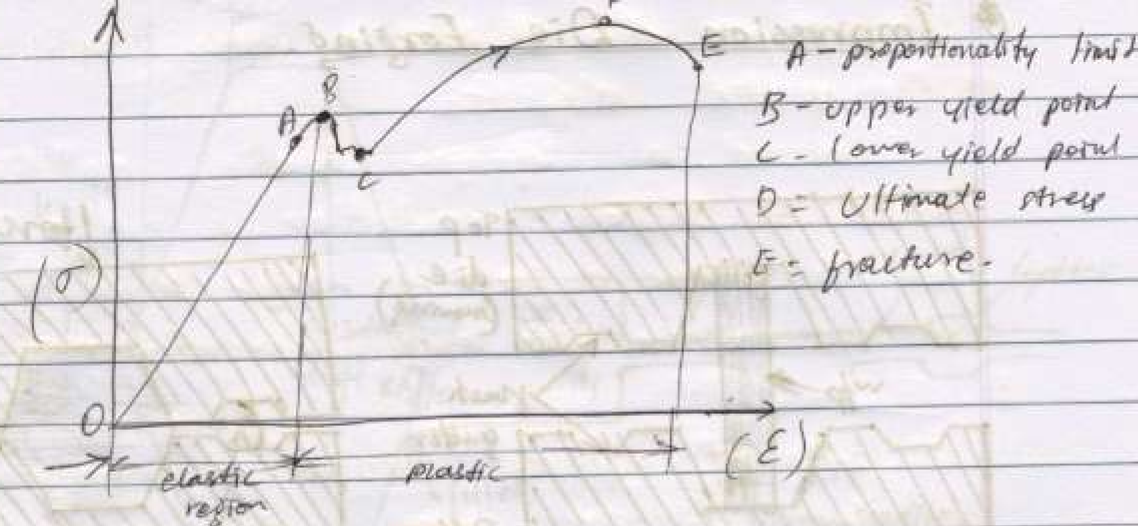
$$\boxed{\varepsilon = \frac{L - L_0}{L_0}}$$

(3) What is flow Stress? Determine with tensile & compression.

Ans Flow stress is defined as the instantaneous value of stress required to continue deforming the material at a constant strain rate in its plastic range. It is the yield strength of the work material as a function of strain.

Determination by Tensile Test.

In a tensile test, a carefully prepared standard specimen is held by its ends in the holding grips of the testing machine. The specimen is subjected to gradually increasing tensile force, while simultaneous observations are made in the elongation of the specimen until the specimen breaks. The stress & strain are classified for each load value, & a graph of stress versus strain is plotted as



Compression Test

In a compression test, higher strains are possible under compressive or upsetting force thereby making the test suitable for most bulk deformation process.

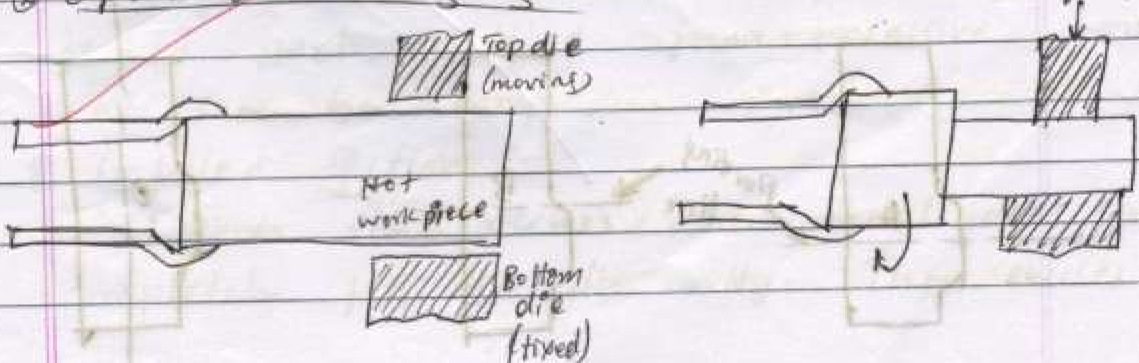
The workpiece is compressed between flat anvils that are smooth & well lubricated to avoid functional effects. The deformation of the work material for every increase in the compressive load is recorded. Compressive stress & strain are calculated & recorded

(5) (a) Define forging & classify forging process:

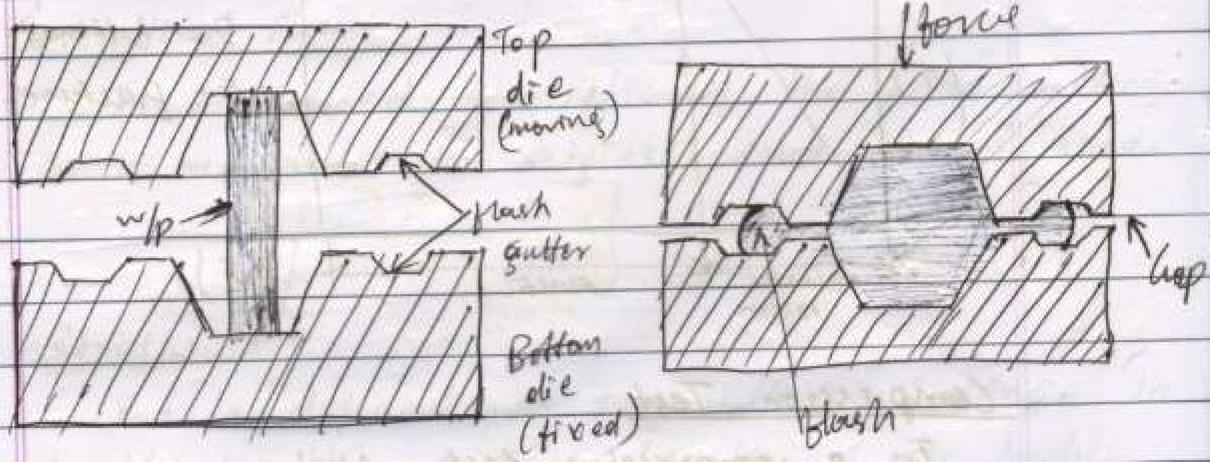
Ans Forging is a type of manufacturing process wherein a metal is heated to its plastic state & then deformed to desired shape & size by the application of compressive forces through a hammer, press or roll ~~etc~~.

Forging Process is broadly classified into 3.

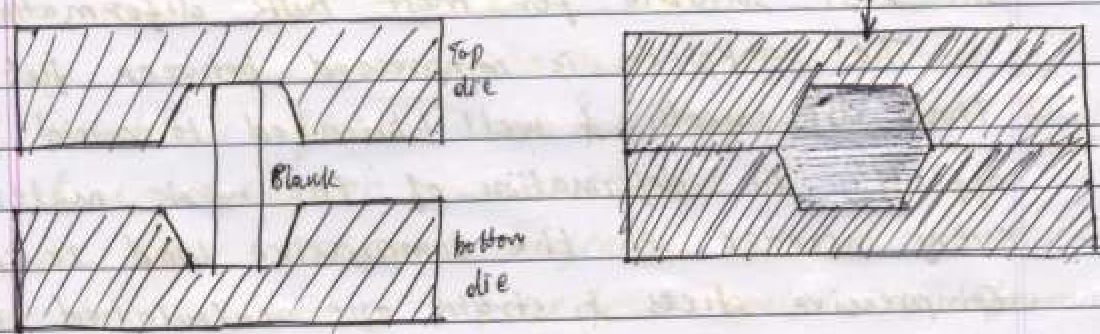
(a) Open die Forging.



(6) Impression Die Forging.



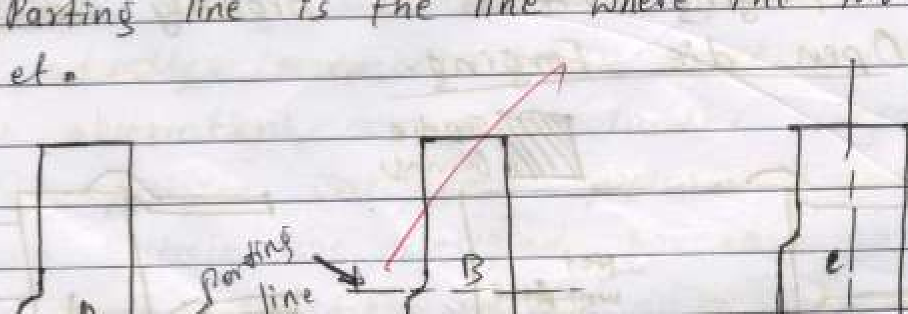
(8) Closed Die forging.

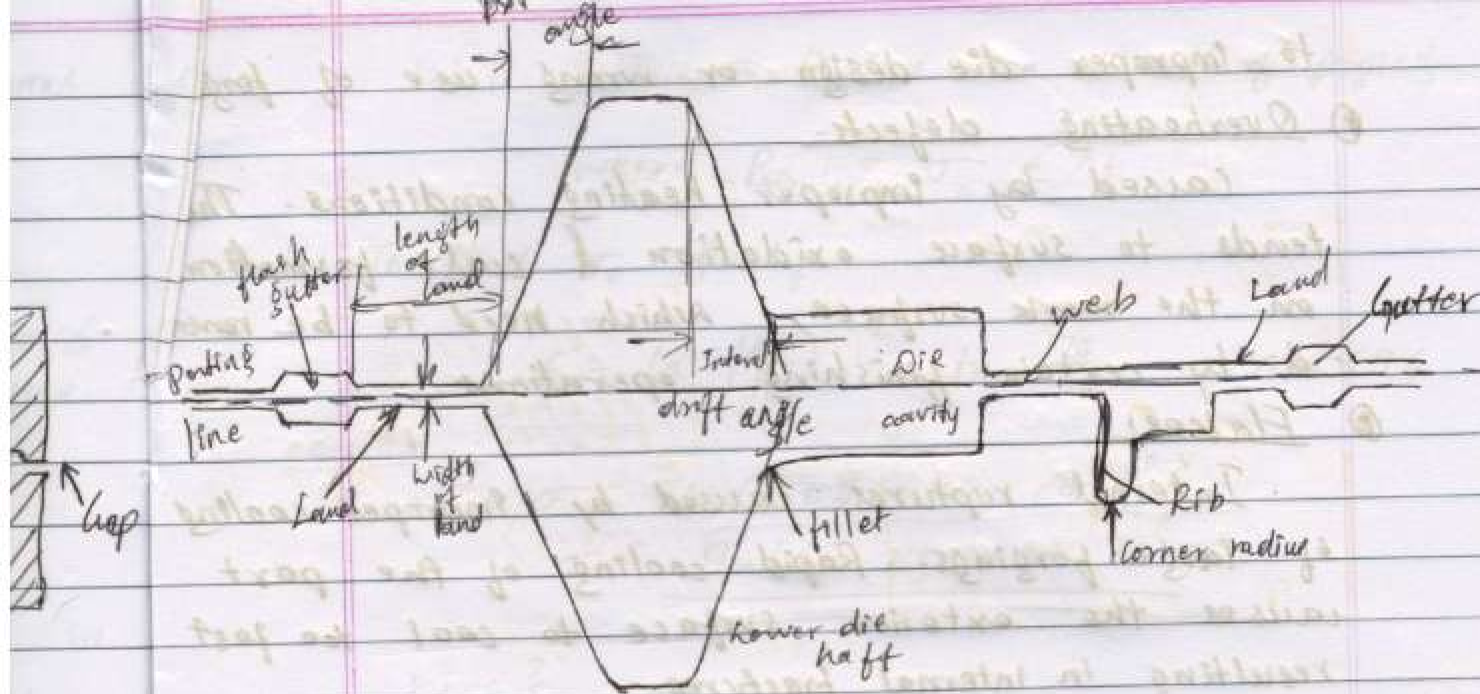


(7) (a) Explain the concept of forging die design parameters.

Ans:

(1) Parting line:
Parting line is the line where the two die halves meet.





① flash. ② Draft ③ Corner & fillet
④ Rib & Web Thickness.
⑤ Other allowances.

7 ⓑ Write a note on forging defects.

Ans ① Cold shut.

It is a discontinuity produced when two surfaces of the metal fold against each other without welding properly.

② Die shift.

Defect resulting from wrong alignment of the die halves. The forged part takes the improper shape resulting in a defect.

③ Surface cracks.

Defects on the work surface of the forged part resulting from excessive working at too low temp.

④ Unfilled section

Defects that occurs when metal does not completely fill the die cavity. This causes due to improper die design or wrong use of forging.

① Overheating defects-

caused by improper heating conditions. This tends to surface oxidation & scale formation on the work surface, which need to be removed by certain finishing operations

⑧ Flakes:

Internal ruptures caused by improper cooling of large forgings. Rapid cooling of the part causes the exterior surface to cool too fast resulting in internal fractures.

⑨ Scale pits.

These are irregular depressions on the surface of the forged part. This is mainly caused due to the improper cleaning of the starting work material being used.

⑥ A circular disc of 150 mm dia & height is 70 mm is forged b/w 2 flat dies to 40 mm height. Find the die load at the end of compression using slab method. Yield strength of material is given by $\sigma = 12(0.02+\epsilon)^{0.05}$ kgf/mm² & $\mu = 0.05$. Also find the mean die pressure.

Solu: Data:

disc dia $d_0 = 150$ mm

height of dis $h_0 = 70$ mm

height after forging $h_f = 40$ mm

$$\sigma = 12(0.02+\epsilon)^{0.05} \text{ kgf/mm}^2$$

W.K.T. $F_{max} = P_{max} \times$ Area at end of forging

where $P_{max} = \sigma_0 \, e^{\frac{2\mu r_f}{h_f}}$

To find σ_0 & r_f

$$\varepsilon = \ln\left(\frac{h_o}{h_f}\right) = \ln \frac{70}{40} = 0.5596$$

$$\sigma = 12(0.02 + 0.5596)^{0.5}$$

$$\sigma = \underline{9.135 \text{ kg/mm}^2 = \sigma_0}$$

$r_f = ?$

Volume before forging = Volume after forging

$$\left(\frac{\pi}{4} d_o^2\right) h_o = \left[\frac{\pi}{4} d_f^2\right] h_f$$

$$150^2 \times 70 = d_f^2 \times 40$$

$$\therefore d_f = 198.4 \text{ mm}$$

$$\boxed{r_f = 99.21 \text{ mm}}$$

Now eqn becomes

$$P_{max} = 9.135 \left[e^{\frac{2 \times 0.05 \times 99.21}{40}}\right]$$

$$P_{max} = \underline{11.706 \text{ kgf/mm}^2}$$

$$\therefore F_{max} = 11.706 \times \left[\frac{\pi}{4} d_f^2\right]$$

$$= 11.706 \times \frac{\pi}{4} \times 198.4^2$$

$$F_{max} = \underline{361.91 \times 10^3} \text{ kgf}$$

forging load $= F_{max} = \underline{361.9 \text{ tonnes}}$

For mean die Pressure. (P_{avg})

$$P_{avg} = \frac{\sigma_0}{2}\left(\frac{h}{\mu r}\right)^2\left[e^{\frac{2\mu r}{h}} - \frac{2\mu r}{h} - 1\right]$$

where $r = r_f$ & $h = h_f$

$$P_{avg} = \frac{9.135}{2}\left[\frac{40}{0.05\times 99.21}\right]^2\left[e^{\frac{198.4\times(0.05)}{40}} - \frac{198.4\times 0.05}{40} - 1\right]$$

$$\boxed{P_{avg} = 9.93\ kgf/mm^2}$$

(5) (b) Write difference b/w forging hammer & Power Press

Forging hammer	Power Press
It derives the power from kinetic energy of a ram & the top portion of die when put into motion.	(1) The energy is delivered through a single continous squeezing reaction which results in uniform deformation.
Here the ram its raised to predetermined height & allowed to drop on the w/p where the PE is controled int KE	No need of Governer of PE to KE becomes of single squeezing action.
Collision takes place b/w upper die & lower die which results in required shape & size	No collision take place b/w upper die & lower die as the force applied is gradu
Reciprocation which of ram completes is operation	Squeezing action of ram completes the operation

10/10

18/20

ALL
OF US
DO **NOT**
HAVE EQUAL
TALENT.

BUT, **ALL**
OF US
HAVE **AN**
EQUAL
OPPORTUNITY
TO DEVELOP
OUR **TALENTS.**

Inspirational
Quotes by
- A.P.J Abdul Kalam

2019-20

INDEX

Name Shivani S.

Standard ~~Section~~ ~~Roll No.~~ /AM16Q104/

Subject AIS.

School / College AMCEC

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Valued

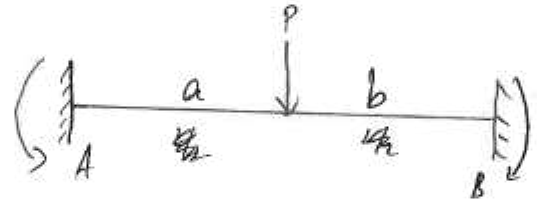
$\frac{10}{10}$

CS

Assignment 1

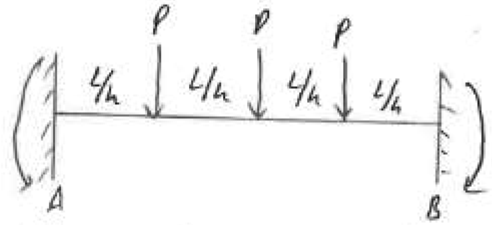
1a. Write the Fixed End Moments for the following Fig 1(a)

a)



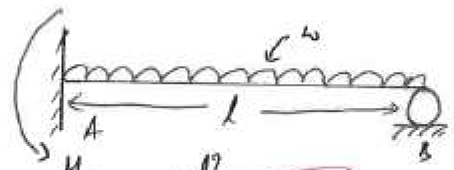
$M_{F_{AB}} = \dfrac{-Pab^2}{l^2}$ $M_{F_{BA}} = \dfrac{Pa^2b}{l^2}$

b)



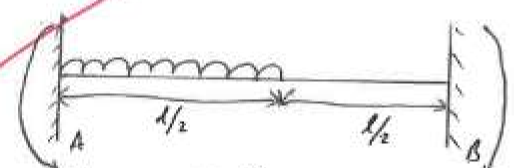
$M_{F_{AB}} = \dfrac{-3PL}{16}$ $M_{F_{BA}} = \dfrac{3PL}{16}$

c)



$M_{F_{AB}} = \dfrac{wl^2}{8}$

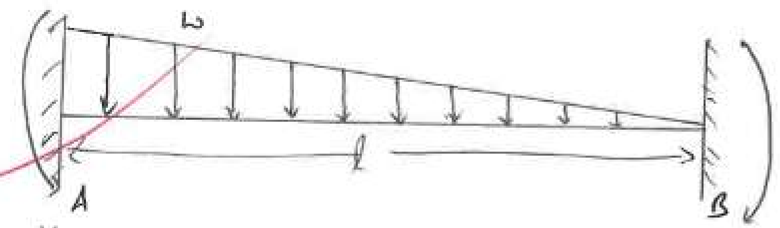
d)



$M_{F_{AB}} = \dfrac{-11wl^2}{192}$ $F_{BA} = \dfrac{11wl^2}{192}$

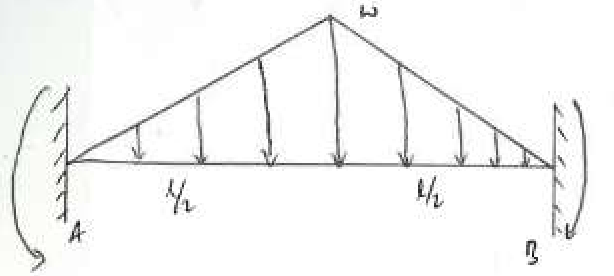
2a. Write the Fixed End Moments for the following Fig 2(a)

a)



$M_{F_{AB}} = -\dfrac{wl^2}{20}$ $M_{F_{BA}} = \dfrac{wl^2}{20}$

b)

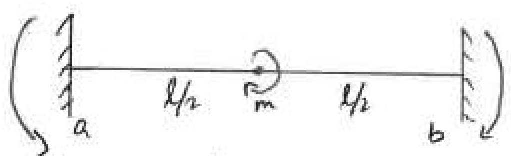


$M_{F_{AB}} = \frac{-5wl^2}{96}$ $\qquad$ $M_{F_{BA}} = \frac{5wl^2}{96}$

c)

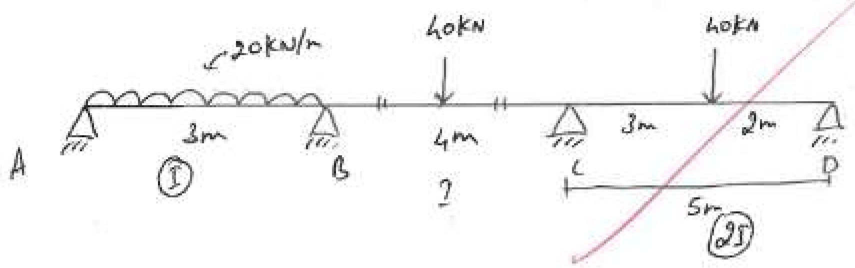
$M_{F_{AB}} = \frac{-6EI\Delta}{l^2}$ $\qquad$ $M_{F_{BA}} = \frac{6EI\Delta}{l^2}$

d)



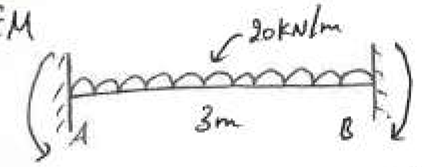
$M_{F_{ab}} = -m \times l/2$ $\qquad$ $M_{F_{ba}} = m \times \frac{l}{2}$

1b) Analyze the continuous beam shown in the fig (b) using SD method and draw BMD.



Step 1: FEM

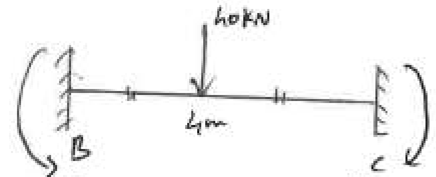
a)



$M_{F_{AB}} = \frac{-wl^2}{12}$ $\qquad$ $M_{F_{BA}} = \frac{wl^2}{12}$

$= -15\,kN\text{-}m$ $\qquad$ $= 15\,kN\text{-}m$

b)

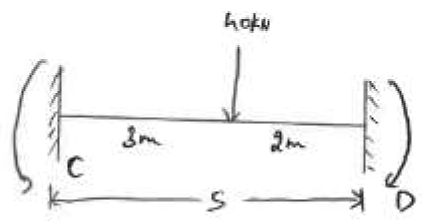


$M_{F_{BC}}$ $\qquad$ $M_{F_{CB}}$

$M_{F_{BC}} = \frac{-wl}{8}$ $\qquad$ $M_{F_{CB}} = \frac{wl}{8}$

$= -20\,kN\text{-}m$ $\qquad$ $= 20\,kN\text{-}m$

c)



$M_{F_{CD}} = \frac{-Wab^2}{l^2}$ $\qquad$ $M_{F_{DC}} = \frac{Wa^2b}{l^2}$

$= \frac{-40 \times 3 \times 2^2}{5^2}$ $\qquad$ $= +\frac{40 \times 3^2 \times 2}{5^2}$

$= -19.2\,kN\text{-}m$ $\qquad$ $= 28.8\,kN\text{-}m$

2: Slope deflection eq^n

for span AB

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)^0$$

$$= -15 + \frac{2EI}{3}(2\theta_A + \theta_B)$$

$$= -15 + 1.33\theta_A EI + 0.67\theta_B EI \quad \text{— (1)}$$

For span BA

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= 15 + 1.33\theta_B EI + 0.67\theta_A EI \quad — (2)$$

For span BC

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -20 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$$= -20 + \theta_B EI + 0.5\theta_C EI \quad — (3)$$

For span CB

$$M_{CB} = M_{F_{CB}} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$= 20 + \theta_C EI + 0.5\theta_B EI \quad — (4)$$

For span CD

$$M_{CD} = M_{F_{CD}} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\delta}{l}\right)$$

$$= -19.2 + \frac{2EI}{5}(2\theta_C + \theta_D)$$

$$= -19.2 + 1.6\theta_C EI + 0.8\theta_D EI \quad — (5)$$

For span DC

$$M_{DC} = M_{F_{DC}} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\delta}{l}\right)$$

$$= 28.8 + 1.6\theta_D EI + 0.8\theta_C EI \quad — (6)$$

Equilibrium Condition

$$\Sigma M_A = 0$$

$$M_{AB} = 0$$

$$-15 + 1.34\theta_A EI + 0.67\theta_B EI = 0 \quad — (7)$$

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$+1.33\theta_B EI + 0.67\theta_A EI - 20 + \theta_B EI + 0.5\theta_C EI = 0$$

$$0.67\theta_A EI + 2.34\theta_B EI + 0.5\theta_C EI - 5 = 0 \quad — (8)$$

$$\Sigma M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$0 + \theta_C EI + 0.5\theta_B EI - 19.2 + 1.6\theta_C EI + 0.8\theta_D EI = 0$$

$$0.5\theta_B EI + 2.6\theta_C EI + 0.8\theta_D EI + 0.8 = 0 \quad — (9)$$

$$\Sigma M_D = 0$$

$$M_{DC} = 0$$

$$28.8 + 1.6\theta_D EI + 0.8\theta_C EI = 0 \quad — (10)$$

add eqn (7) & (9), (8) & (10)

$$\theta_A EI + 0.67\theta_B EI + 0.5\theta_B EI + 2.6\theta_C EI + 0.8\theta_D EI - 5 = 0$$

$$I + 1.17\theta_B EI + 2.6\theta_C EI + 0.8\theta_D EI - 20 = 0 \quad — (11)$$

$$+ 2.34\theta_B EI + 0.5\theta_C EI - 5 + 28.8 + 1.6\theta_D EI + 0.8\theta_C EI = 0$$

$$+ 2.34\theta_B EI + 1.38\theta_C EI + 1.6\theta_D EI + 28.3 = 0 \quad — (12)$$

with 2 & Subtitution with eqn (12)

$$8\theta_A EI + 2.34\theta_B EI + 5.2\theta_C EI + 1.6\theta_D EI - 40 = 0$$

$$7\theta_A EI - 2.34\theta_A EI - 1.38\theta_C EI - 1.6\theta_D EI - 28.8 = 0$$

$$01\theta_A EI + 3.82\theta_C EI - 68.8 = 0 \quad — (13)$$

by eq^n 11, 8, 12

$\theta_A = 13.18/EI$

$\theta_B = -3.97/EI$

$\theta_C = 10.94/EI$

by eq^n 9

$\theta_D = -34.07/EI$

Step 4: Sub in eq^n 1 to 6

$M_{AB} = 0.0013$ KN-m

$M_{BA} = 18.51$ KN-m

$M_{BC} = -18.5$ KN-m

$M_{CB} = 28.95$ KN-m

$M_{CD} = -28.95$ KN-m

$M_{DC} = -16.96$ KN-m

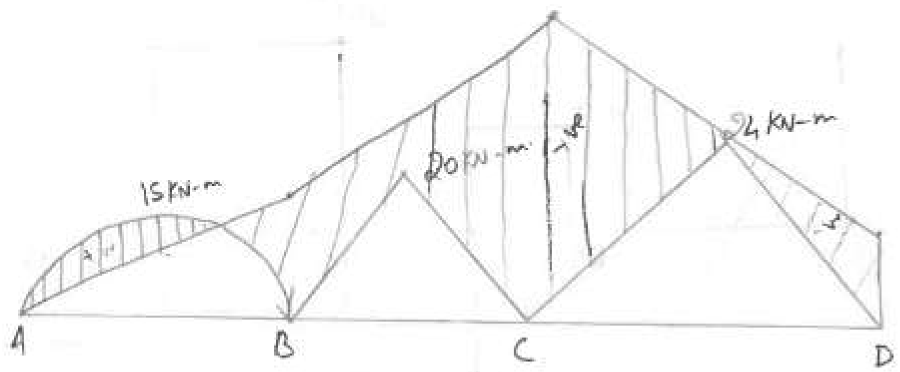
Step 5: Max BM

$\frac{wl^2}{12} = \frac{20\times3^2}{12} = 15$ KN-m

$\frac{wl}{8} = \frac{40\times4}{8} = 20$ KN-m

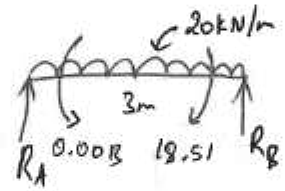
$\frac{wa}{l} = \frac{40\times3}{5} = \cancel{120KN} = 24$ KN-m

Step 6: BMD



Step 7: Reaction

a)



$\Sigma M_A = 0$

$-R_B\times3 + 20\times3\times\frac{3}{2} + 18.51 = 0$

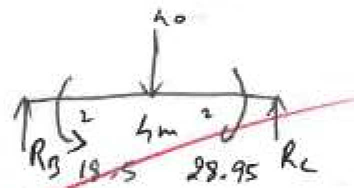
$R_B = 36.17$ KN

$\Sigma M_B = 0$

$R_A\times3 - 20\times3\times\frac{3}{2} + 18.51 = 0$

$R_A = 23.83$ KN

b)



$\Sigma M_B = 0$

$-R_C\times4 + 40\times2 - 18.5 + 28.95 = 0$

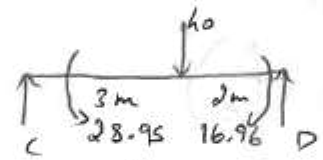
$R_C = 22.61$ KN

$\Sigma M_C = 0$

$R_B\times4 - 40\times2 - 18.5 + 28.95 = 0$

$R_B = 17.38$ KN

c)



ΣM_D

$R_C\times5 - 40\times2 + 28.95 + 16.96 = 0$

$R_C = \cancel{6.81}\ 25.18$ KN

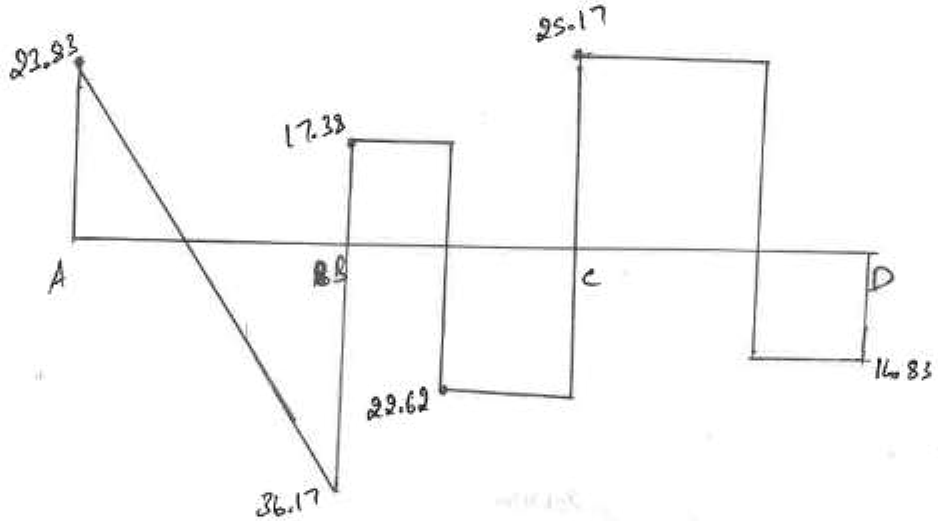
ΣM_C

$-R_D\times5 + 40\times2 + 28.95 - 16.96 = 0$

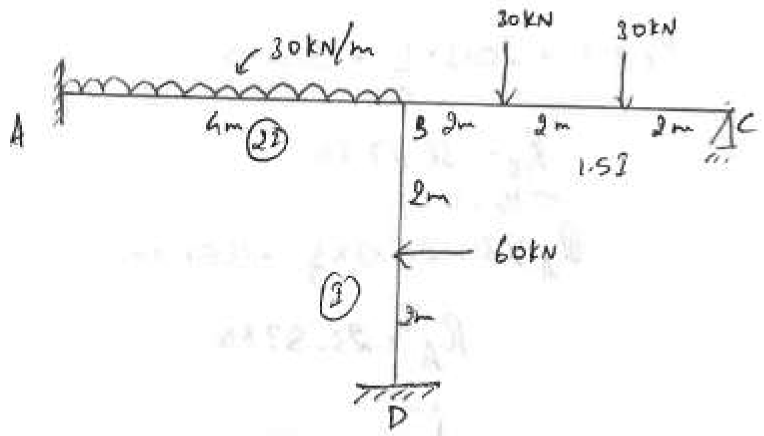
$R_D = 6.81$ KN

Step 8:

$R_A = 23.83$
$R_B = 53.55$
$R_C = 42.79$
$R_D = 6.81$

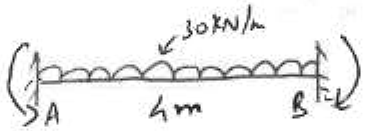


2b. Analyze the frame shown in fig 2(b) by SD Method. Draw BMD for the same



Step 1: FEM

a)

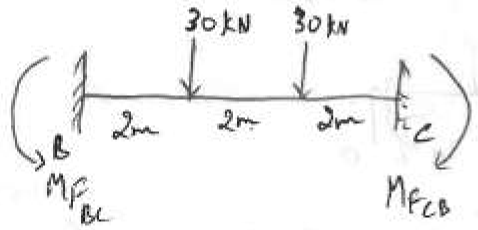


$MF_{AB} = -\frac{wl^2}{12}$ $\qquad M_{F_{BA}} = \frac{wl^2}{12}$

$= \frac{-30\times 4^2}{12}$ $\qquad = \frac{30\times 4^2}{12}$

$= -40kN\text{-}m$ $\qquad = 40kN\text{-}m$

b)

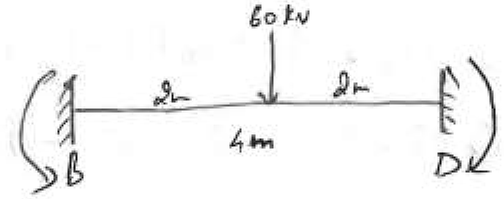


$M_{F_{BC}} = -\frac{W_1ab^2}{l^2} - \frac{W_2ab^2}{l^2}$ $\qquad MF_{CB} = \frac{W_1a^2b}{l^2} + \frac{W_2a^2b}{l^2}$

$= -\frac{30\times 2^2}{6} - \frac{30\times 2\times 2^2}{6}$ $\qquad = \frac{30\times 2^2\times 2}{6} + \frac{30\times 2^2\times 2}{6}$

$= -80\ kN\text{-}m$ $\qquad = 80\ kN\text{-}m$

c)



$M_{F_{BD}} = -\frac{wl}{8} = -30\ kN\text{-}m$ $\qquad M_{F_{DB}} = \frac{wl}{8} = 30kN\text{-}m$

Step 2: Slope deflection

$M_{AB} = M_{F_{AB}} + \frac{2\times 2EI}{l^2}\left(2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{l}}^{0}\right)$

$= -40 + \frac{4EI}{\cancel{16}\,4}(\theta_B)$

$= -40 + \cancel{0.25}\,\theta_B\ EI$ — ①

$M_{BA} = M_{F_{BA}} + \frac{4EI}{\cancel{16}\,4}\left(2\theta_B + \cancel{\theta_A}^{0} - \cancel{\frac{3\delta}{l}}^{0}\right)$

$= 40 + \cancel{0.2}\,2EI\theta_B$ — ②

$M_{BC} = M_{F_{BC}} + \frac{1.5\times 2EI}{6}\left(2\theta_B + \theta_C - \cancel{\frac{3\delta}{l}}^{0}\right)$

$= -80 + \theta_B EI + 0.5EI\theta_C$ — ③

$M_{CB} = 80 + \theta_C EI + 0.5EI\theta_B$ — ④

$M_{BD} = M_{F_{BD}} + \frac{2EI}{4}\left(2\theta_B + \cancel{\theta_D}^{0} - \cancel{\frac{3\delta}{l}}^{0}\right)$

$= -30 + \theta_B EI$ — ⑤

$M_{DB} = 30 + 0.5\theta_B EI$ — ⑥

Step3: Equilibrium Condition

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$40 + 2EI\theta_B - 80 + \theta_B EI + 0.5EI\theta_C - 30 + \theta_B EI = 0$$

$$-70 + 4\theta_B EI + 0.5EI\theta_C = 0 \quad — (1)$$

$$\Sigma M_C = 0$$

$$M_{CB} = 0$$

$$80 + 0.5EI\theta_B + \theta_C EI \quad — (8)$$

$$\theta_B = 29.33/EI$$

$$\theta_C = -94.67/EI$$

Step4: Sub θ in eq^n (1) to (8)

$M_{AB} = -10.67 kN\text{-}m$

$M_{BA} = 98.66 kN\text{-}m$

$M_{BC} = -98 \ kN\text{-}m$

$M_{CB} = -0.005 \ kN\text{-}m \simeq 0$

$M_{BD} = -0.67$

$M_{DB} = 44.66$

Step5: Max BM

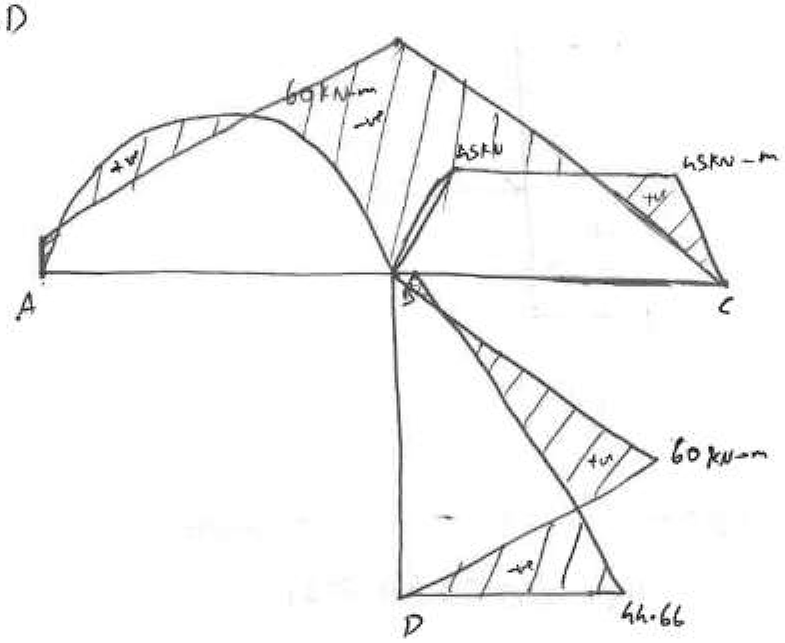
$$\frac{wl^2}{8} = \frac{30\times 4^2}{8} = 60 kN\text{-}m$$

$$\frac{wl}{4} \ \frac{60\cdot 4}{4} = 45 kN\text{-}m$$

$$\frac{wl}{4} = 45 kN\text{-}m$$

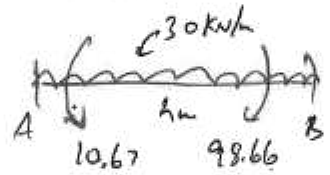
$$\frac{wl}{4} = \frac{60\times 4}{4} = 60 kN\text{-}m$$

Step6: BMD



Step7: Reaction

a)



$$\Sigma M_A = 0$$

$$-R_B\times 4 + 30\times 4\times\frac{4}{2} - 10.67 + 98.66 = 0$$

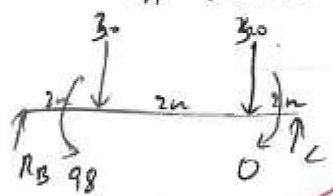
$$R_B = 81.99 kN \simeq 82$$

$$\Sigma M_B = 0$$

$$R_A\times 4 - 30\times 4\times\frac{4}{2} - 10.67 + 98.66 = 0$$

$$R_A = 38 kN$$

b)



$$\Sigma M_B = 0$$

$$-R_C\times 6 + 30\times 4 + 30\times 2 - 98 = 0$$

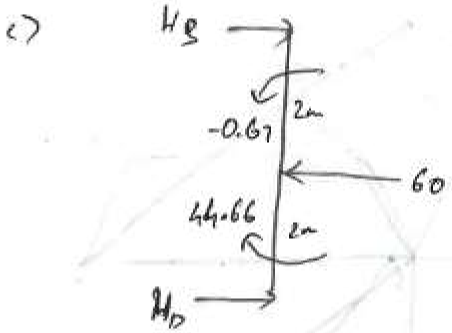
$$R_C = 13.67 kN$$

$$\Sigma M_C = 0$$

$$R_B\times 6 - 30\times 4 - 30\times 2 - 98 = 0$$

$$R_B = 46.33 kN$$

(c)



$\Sigma M_B = 0$

$-H_D \times 4 + 60 \times 2 - 0.67 + 44.66 = 0$

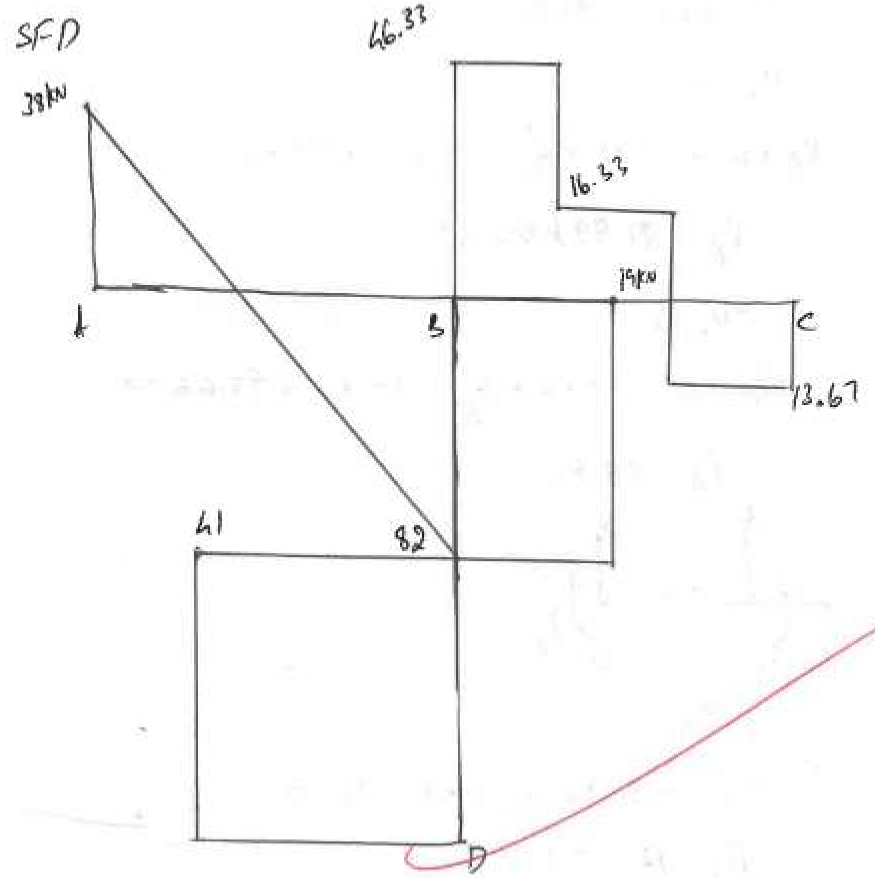
$H_D = 40.99\ KN \cong 41$

$\Sigma M_D = 0$

$H_B \times 4 - 60 \times 2 - 0.67 + 44.66 = 0$

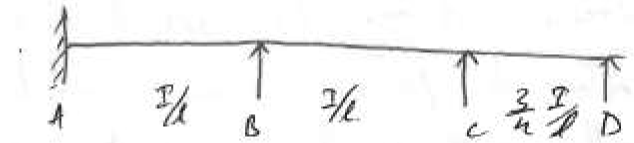
$H_B = 19\ KN$

Step 3: SFD



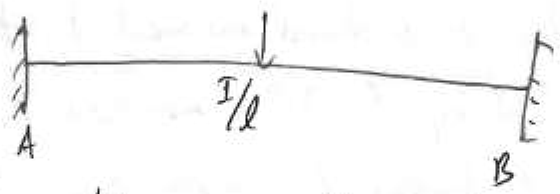
3a. Give the relative Stiffness.

(i) If far end is fixed or continuous support.

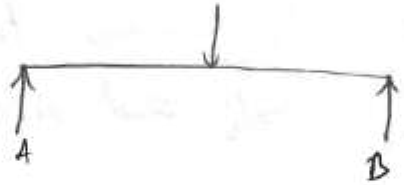


Relative stiffness
BC (Continuous to Support) $= \frac{I}{l}$

(ii) far end is not continuous support

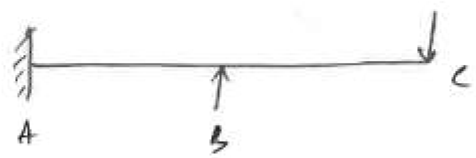


Relative stiffness AB $= \frac{I}{l}$ (fixed)



Relative stiffness AB $= \frac{3}{4}\frac{I}{l}$

(iii) Overhanging



Relative Stiffness BC = 0

4a. Discuss about Stiffness, carryover factor with diagram

Stiffness factor:

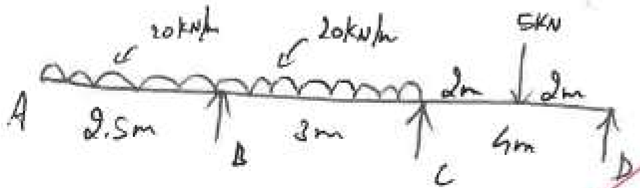
* It is the moment that must be applied at one end of a constant section member to produce a unit rotation of that end when the other end is fixed, i.e. $K = 4EI/l$.

* It is the moment required to rotate the near end of a prismatic member through a unit angle without translation, the far end being hinged is $K = 3EI/l$.

Carry Over Factor:

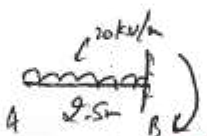
It is the ratio of induced moment to the applied moment. The carry over factor is always (1/2) for members of constant moment of inertia. If the end is hinged/pin connected, the carry over factor is zero. It should be mentioned here that carry over factors value differ for non-prismatic members. For non-prismatic beams; the carry over factor is not half and is different for both ends.

3b.



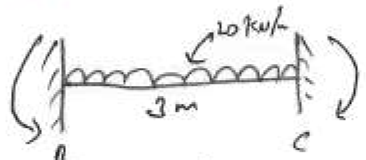
FEM:

a)



$M_{AB} = 20 \times 2.5 = 50 kN\text{-}m$

b)



$M_{BC} = -\frac{wl^2}{12}$ $= -15 kN\text{-}m$

$M_{CB} = \frac{wl^2}{12}$ $= +15 kN\text{-}m$

c)

$M_{FCD} = -\frac{wl}{8}$ $= -2.5 kN$

$M_{FDC} = \frac{wl}{8}$ $= 2.5 kN$

Step 2:

Relative Stiffness,

$BC = \frac{3}{4}\frac{I}{l} = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = 0.25$

$CD = \frac{3}{4}\frac{I}{l} = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16} = 0.19$

Step 3: Distribution table,

Joints	Members	Relative stiffness	ΣK_i	$df = \frac{K}{\Sigma K_i}$
C	CB	$\frac{1}{4}$ 0.25	0.44	0.57
	CD	$\frac{3}{16}$ 0.19		0.43

Step 4: Final MD table.

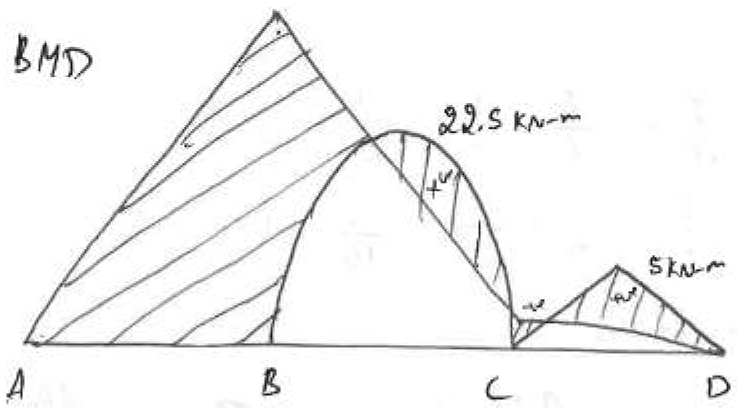
Joint		B	C		D
Members	BA	BC	CB	CD	DC
D.F	-	-	0.57	0.43	-
FEM	50	-15	15	-2.5	2.5
Bal	-	-35	-7.12	-5.37	-2.5
C/O	-	-	-17.5	1.25	-
Bal	-	-	10.69	8.06	
C/O	-	-	-	-	-
Bal	-	-	-	-	-
	50	-50	1.07	-1.06	0

Steps: Max BM

$BC = \frac{wl^2}{8} = \cancel{15 KN-m}\ 22.5 KN-m$

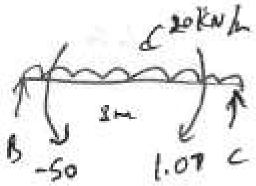
$CD = \frac{wl}{4} = \frac{5 \times 4}{4} = 5 KN-m$

Step 6: BMD



Step 7: Reactions

a)



$\Sigma M_B = 0$

$-R_C \times 3 + 20 \times 3 \times \frac{3}{2} - 50 + 1.07 = 0$

$R_C = 13.69 KN$

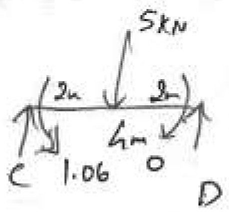
$\Sigma M_C = 0$

$R_B \times 3 - 20 \times 3 \times \frac{3}{2} - 50 + 1.07 = 0$

$R_B = 46.31 KN$

$R_B = 46.31$
$R_C = 16.45$
$R_D = 2.23$

b)



$\Sigma M_C = 0$

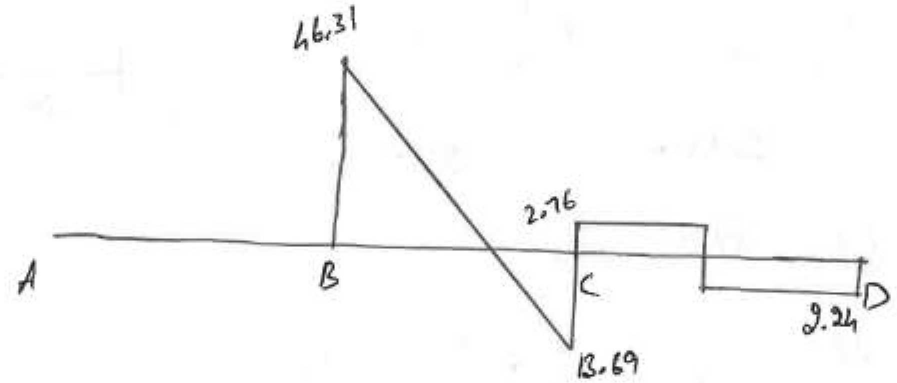
$-R_D \times 4 + 5 \times 2 - 1.06 = 0$

$R_D = 2.23 KN$

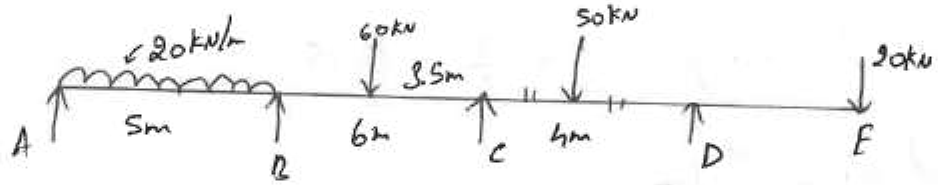
$\Sigma M_D = 0$

$R_C \times 4 - 5 \times 2 - 1.06 = 0$ $R_C = 2.76 KN$

Step 8: SFD

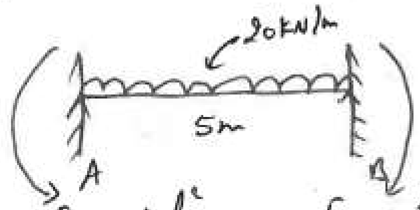


4b)



Step 1: FEM

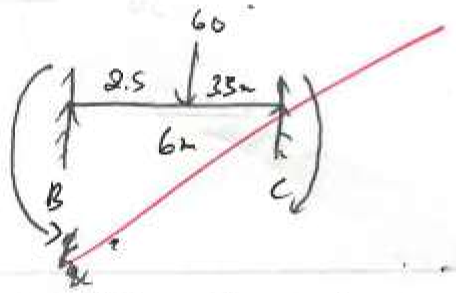
a)



$F_{AB} = -\frac{wl^2}{12}$ $F_{BA} = \frac{wl^2}{12}$

$= -41.67$ $= 41.67$

b)

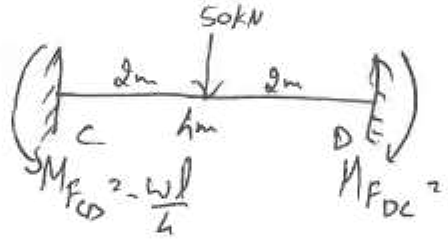


~~$M_{F_{BC}} = \frac{-w_1 ab^2}{l^2} - \frac{w_2 ab^2}{l^2}$ $M_{F_{CB}} = \frac{+w_1 a^2 b}{l^2} + \frac{w_2 a^2 b}{l^2}$~~

$M_{F_{BC}} = \frac{-Pab^2}{l^2}$ $M_{F_{CB}} = \frac{Pa^2b}{l^2}$

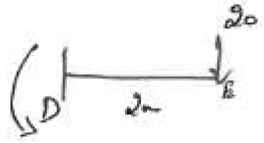
$= -51.8 KN-m$ $M_{F_{CB}} = 36.46 KN-m$

c)



$M_{F_{CD}} = -\frac{wl}{4}$ $= -25$ kN-m

$M_{F_{DC}} = \frac{wl}{4}$ $= 25$ kN-m



$M_{D_E} = -40$ kN

5.77

Step 2: Relative Stiffness

$AB = \frac{3}{4}\frac{I}{l} = \frac{3}{4} \times \frac{1}{5} = 0.15$

$BC = \frac{I}{l} = \frac{1}{6} = 0.17$

$CD = \frac{3}{4} \times \frac{I}{l} = \frac{3}{4} \times \frac{1}{4} = 0.19$

Step 3: Distribution Factors

Joints	Members	Relative Stiffness	ΣK_i	$DF = \frac{K}{\Sigma K_i}$
B	BA	0.15	0.32	0.47
	BC	0.17		0.53
C	CB	0.17	0.36	0.47
	CD	0.19		0.53

Step 4: Final MD table

9.31 24.01 13.56 2.53 11.46

Joint	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	DE
D F	–	0.47	0.53	0.47	0.53	–	
FEM	-41.67	41.67	-51.	36.46	-25	25	-40
Bal	41.67	4.38	4.94	5.38	6.07	+15	–
C/O	–	20.83	2.18	3.03	2.69	–	–
Bal	–	-11.28	-12.72	[illegible]	[illegible]	–	–
C/O	–	–	0.59	-6.36	–	–	–
Bal	–	-0.27	-0.31	2.99	3.37		–
C/O	–	–	1.49	-0.15	–	–	–
Bal	–	-0.7	-0.79	0.07	0.078	–	–
C/O	–	–	0.035	-0.39	–	–	–
Bal	–	-0.016	-0.018	0.18	0.2	–	–
C/O	–	–	0.09	-0.009	–	–	–
Bal	–	-0.04	-0.05	+0.0042	0.004	–	–
	0	54.57	-54.56	42.81	-42.83	40	-40

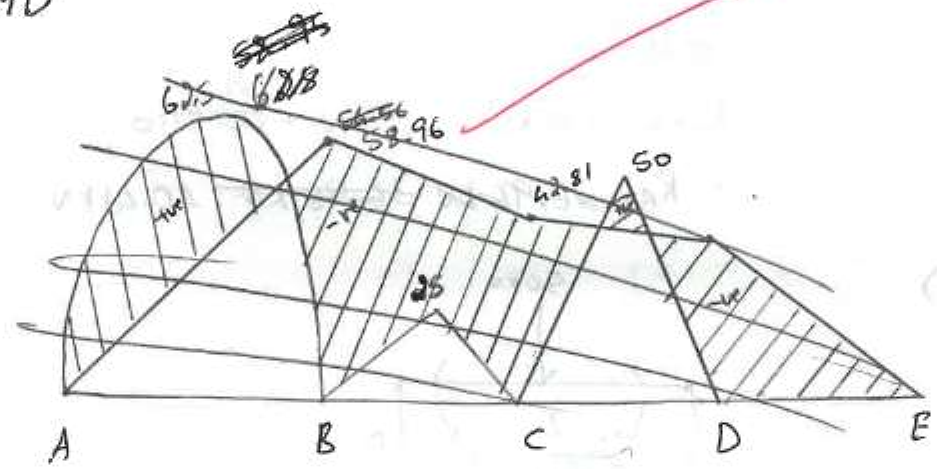
Step 5: Max BM,

a) $\frac{wl^2}{8} = 62.5$

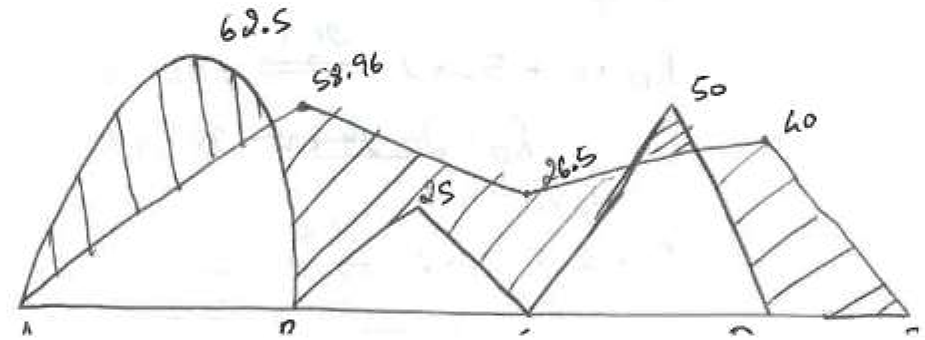
b) $\frac{Wa}{l} = 25$

c) $\frac{wl}{4} = 50$

Step 6: BMD

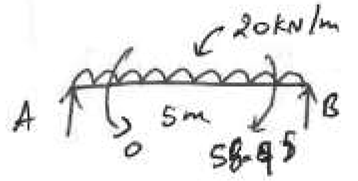


5.99



Step 7:

a)



$\Sigma M_A = 0$

$-R_B \times 5 + 20 \times 5 \times \frac{5}{2} + 58.95 = 0$

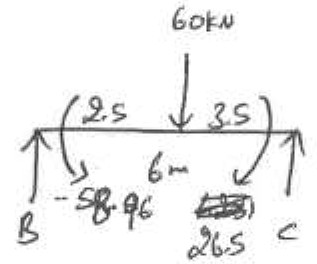
$R_B = \cancel{60.41kN}\ 61.79\,kN$

$\Sigma M_B = 0$

$R_A \times 5 - 20 \times 5 \times \frac{5}{2} + 58.95 = 0$

$R_A = \cancel{39.08\,kN}\ 38.21\,kN$

b)



$\Sigma M_B = 0$

$-R_C \times 6 + 60 \times 2.5 - 58.96 + \cancel{42.81}\ 26.5 = 0$

$R_C = \cancel{23.04\,kN}\ 19.59\,kN$

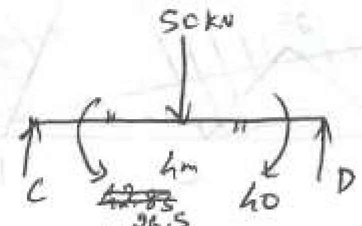
$\Sigma M_C = 0$

$R_B \times 6 - 60 \times 3.5 - 58.96 + \cancel{42.81}\ 26.5 = 0$

$R_B = \cancel{26.96\,kN}\ \cancel{36.96\,kN}\ 40.41\,kN$

~~$R_A = 39.08$~~
~~$R_B = 97.37$~~
~~$R_C = 48.74$~~
~~$R_D = 24.29$~~

c)



$\Sigma M_C = 0$

$-R_D \times 4 + 50 \times 2 - \cancel{42.83}\ 26.5 + 40 = 0$

$R_D = \cancel{24.29\,kN}\ 28.37\,kN$

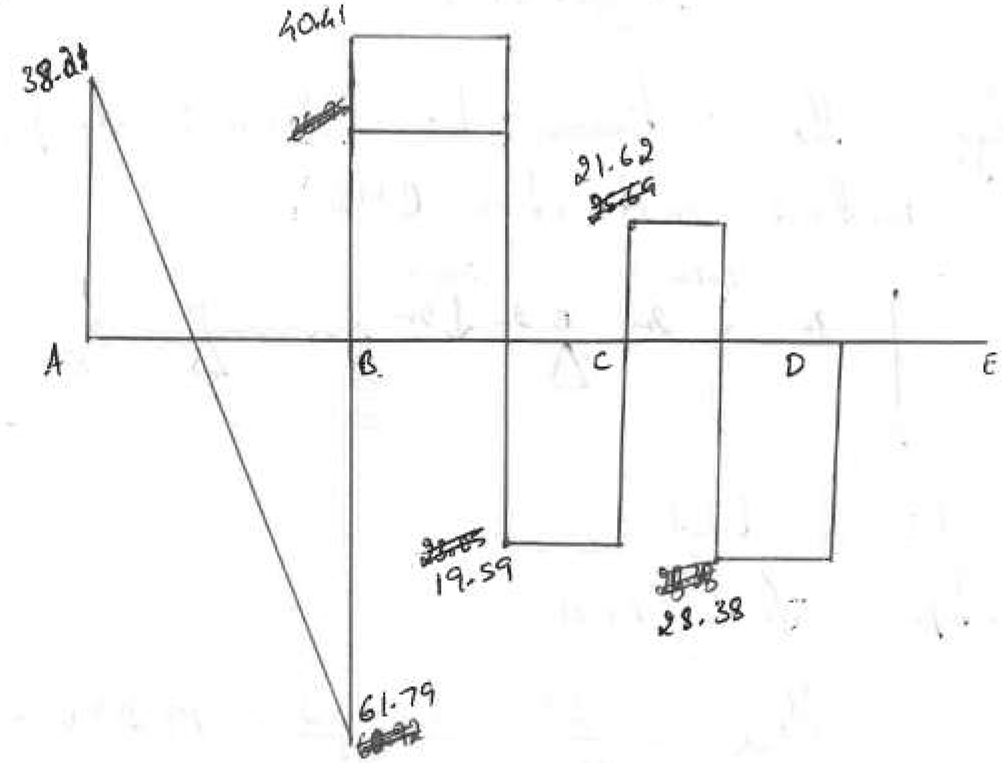
$\Sigma M_D = 0$

$R_C \times 4 - 50 \times 2 - \cancel{42.83}\ 26.5 + 40 = 0$

$R_C = \cancel{25.71\,kN}$ [illegible] 21.63 kN

$R_A = 38.21$
$R_B = 102.2$
$R_C = 41.21$
$R_D = 28.37$

Step 8: SFD



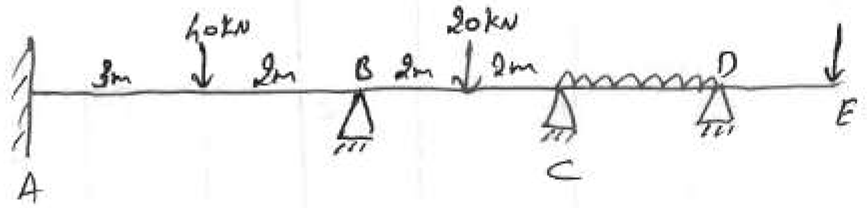
Step 6: Final MD table

18.14
9.97

Joint	A	B		C		D	
Members	AB	BA	BC	CB	CD	DC	DE
D.F	–	0.47	0.53	0.47	0.53	–	–
FEM	-41.67	41.67	-51	36.46	-25	25	-40
Bal	41.67	4.38	4.94	-5.38	-6.07	15	–
c/o	–	20.83	-2.69	2.47	7.5	–	–
Bal	–	-8.52	-9.61	-4.68	-5.28	–	–
c/o	–	–	-2.34	-4.8	–	–	–
Bal	–	1.09	1.24	2.25	2.54	–	–
c/o	–	–	1.12	0.62	–	–	–
Bal	–	-0.53	-0.59	-0.29	-0.33	–	–
c/o	–	–	-0.14	-0.29	–	–	–
Bal	–	+0.065	+0.07	0.13	0.15	–	–
c/o	–	–	0.065	0.035	–	–	–
Bal	–	-0.03	-0.03	-0.02	-0.02	–	–
	0	58.95	-58.96	26.5	-26.5	40	-40

Assignment 2

1. Analyse the continuous beam shown in the fig. 1 using MD method and draw BMD



EI Constant

Sol: Step 1: Finding FEM

$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-40\times3\times2^2}{5^2} = -19.2\ kN\text{-}m$$

$$M_{FBA} = \frac{wa^2b}{l^2} = \frac{40\times3^2\times2}{5^2} = 28.8\ kN\text{-}m$$

$$M_{FBC} = \frac{-wl}{8} = \frac{-20\times4}{8} = -10\ kN\text{-}m$$

$$M_{FCB} = \frac{wl}{8} = \frac{20\times4}{8} = 10\ kN\text{-}m$$

$$M_{FCD} = \frac{-wl^2}{12} = \frac{-10(3)^2}{12} = -7.5\ kN\text{-}m$$

$$M_{FDC} = \frac{wl^2}{12} = \frac{10(3)^2}{12} = 7.5\ kN\text{-}m$$

$$M_{FDE} = -10\times2 = -20\ kN\text{-}m$$

Step 2: Relative Stiffness:

$K_{AB} = I/l = 1/5$ $\quad$ $K_{BC} = I/l = 1/4$

$K_{CD} = \frac{3}{4}\frac{I}{l} = \frac{1}{4}$

Step 3: Distribution table

Joint	Members	K	ΣK	$D = \frac{K}{\Sigma K}$
B	BA	1/5	9/20	0.44
	BC	1/4		0.55
C	CB	1/4	1/2	0.5
	CD	1/4		0.5

Step 4: MD table

Joint	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	DE
D.F	–	0.44	0.55	0.5	0.5	–	–
FEM	-19.2	28.8	-10	10	-7.5	7.5	-20
Bal	–	-8.27	-10.30	-1.25	-1.25	12.5	
C/O	-4.135	0	-0.62	-5.15	6.25	–	–
Bal	–	0.27	0.34	-0.55	-0.55	–	–
C/O	0.13	0	0.27	-0.17	–	–	–
Bal	–	-0.11	-0.14	-0.08	-0.08	–	–
C/O	-0.05	0	-0.04	-0.07		–	–
	-23.25	20.69	-20.19	3.17	-3.13	20	-20

Step 5: Final Moments

$M_{AB} = -23.25\ kN\text{-}m$ $\quad$ $M_{DC} = 20\ kN\text{-}m$

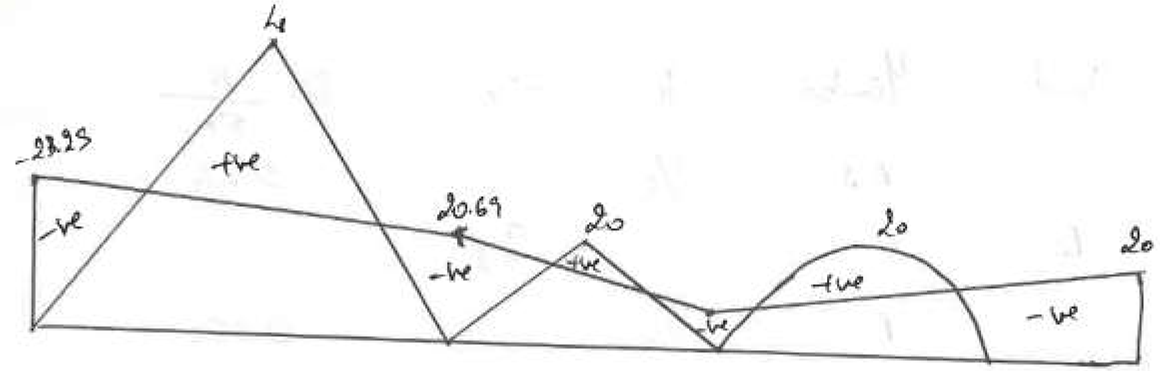
$M_{BA} = 20.69\ kN\text{-}m$ $\quad$ $M_{DE} = -20\ kN\text{-}m$

$M_{BC} = -20.19\ kN\text{-}m$

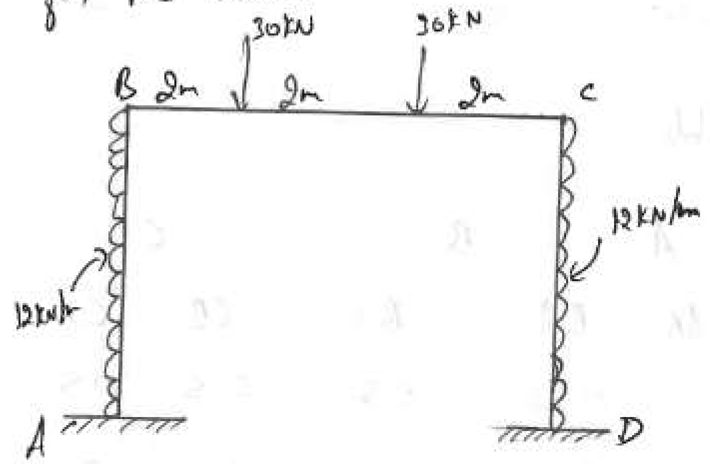
$M_{CB} = 3.17\ kN\text{-}m$

$M_{CD} = -3.13\ kN\text{-}m$

Step 6: BMD:



2 Analyze the frame shown in fig. 2 by MD method. Draw BMD for the same



Step 1: FEM

$M_{FAB} = \frac{-wl^2}{12} = \frac{-12\times4^2}{12} = -16$ kN-m

$M_{FBA} = \frac{wl^2}{12} = \frac{12\times4^2}{12} = 16$ kN-m

$M_{FBC} = \frac{-2Pl}{9} = \frac{-2\times30\times6}{9} = -40$ kN-m

$M_{FCB} = \frac{2Pl}{9} = \frac{2\times30\times6}{9} = 40$ kN-m

$M_{FCD} = \frac{-wl^2}{12} = \frac{-12\times4^2}{12} = -16$ kN-m

$M_{FDC} = \frac{wl^2}{12} = \frac{12\times4^2}{12} = 16$ kN-m

Step 2: Relative stiffness

$K_{AB} = I/l = 1/4$

$K_{BC} = I/l = 1.5/6 = 1/4$

$K_{CD} = I/l = 1/4$

Step 3: Distribution table

Joint	Member	K	ΣK	$D = \frac{K}{\Sigma K}$
B	BA	1/4	1/2	0.5
	BC	1/4		0.5
C	CB	1/4	1/2	0.5
	CD	1/4		0.5

Step 4: MD table

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.5	0.5	0.5	0.5	-
FEM	-16	16	-40	40	-16	16
Bal	-	12	12	-12	-12	-
C/o	6	0	-6	6	0	-6
Bal	-	3	3	-3	-3	-
C/o	1.5	0	-1.5	1.5	0	-1.5
Bal	-	0.75	0.75	-0.75	-0.75	-
C/o	0.37	0	-0.37	0.37	0	-0.37
	8.13	31.75	-32.1	32.1	-31.75	8.13

Step 5: Final Moments

$M_{AB} = -8.13$ kN-m

$M_{BA} = 31.75$ kN-m

$M_{BC} = -32.1$ kN-m

$M_{CB} = 32.1$ kN-m

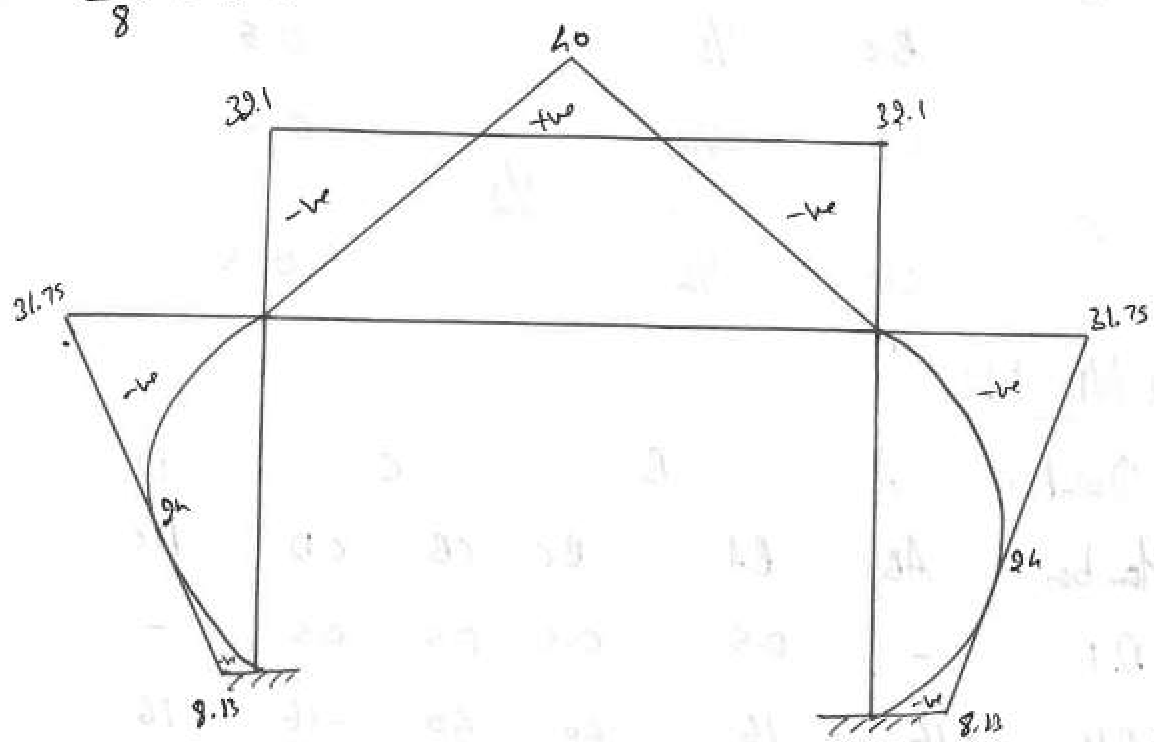
$M_{CD} = -31.75$ kN-m

$M_{DC} = 8.13$ kN-m

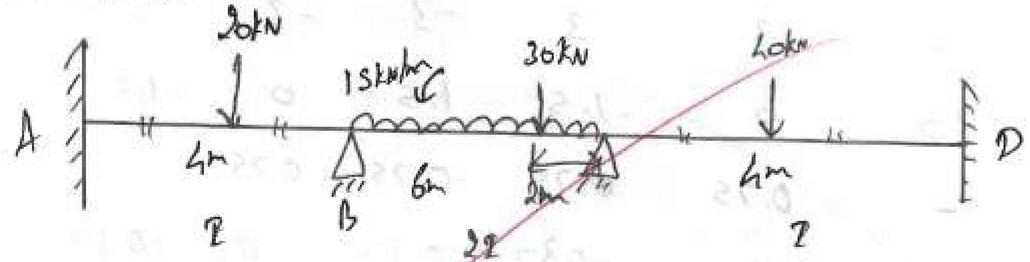
Step6: BMD: $\frac{wl^2}{8} = 24\ kN$

$\frac{Wab}{l} = 40kN$

$\frac{wl^2}{8} = 24\ kN$



3. Analyze the beam shown in fig.3 using Kani's method draw BMD



Step1: FEM:

$M_{FAB} = \frac{-wl}{8} = \frac{-20\times4}{8} = -10kN\text{-}m$

$M_{FBA} = \frac{wl}{8} = \frac{20\times4}{8} = 10kN\text{-}m$

$M_{FBC} = \frac{-wl^2}{12} - \frac{Wab^2}{l^2} = \frac{-15\times6^2}{12} - \frac{30\times4\times2^2}{6^2} = -58.3kN\text{-}m$

$M_{FCB} = \frac{wl^2}{12} + \frac{Wa^2b}{l^2} = \frac{15\times6^2}{12} + \frac{30\times4^2\times2}{6^2} = 71.66\ kN\text{-}m$

$M_{FCD} = \frac{-wl}{8} = \frac{-40\times4}{8} = -20kN\text{-}m$

$M_{FDC} = \frac{wl}{8} = \frac{40\times4}{8} = 20kN\text{-}m$

Step2: Relative stiffness

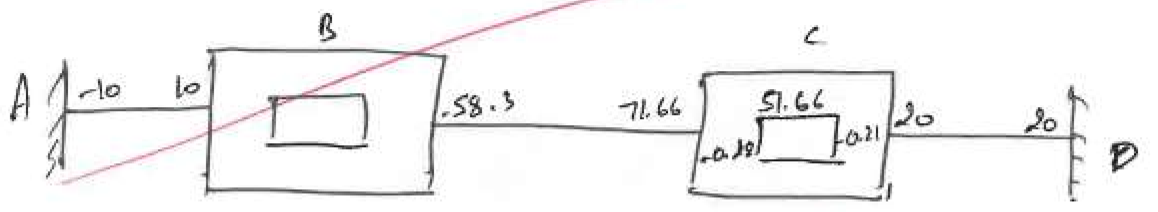
$K_{AB} = I/l = 1/4$

$K_{BC} = I/l = 2/6 = 1/3$

$K_{CD} = I/l = 1/4$

Step3: Rotation factor

Joint	Member	K	ΣK	d	RF
B	BA	1/4	7/12	0.42	-0.21
	BC	1/3		0.57	-0.28
C	CB	1/3	7/12	0.57	-0.28
	CD	1/4		0.42	-0.21



0	10.163	13.524	-18.251	-13.688	0
0	13.975	18.634	-19.682	-14.761	0
0	14.276	19.034	-19.794	-14.845	0
0	14.299	19.066	-19.803	-14.852	0

-10	10	-58.3	71.66	-20	20
0	14.299	19.066	-19.803	-14.852	0
0	14.299	19.066	-19.803	-14.852	0
14.299	0	-19.803	19.066	0	-14.852

Step: Final Moments

$M_{AB} = 4.299$ kN-m

$M_{BA} = 38.598$ kN-m

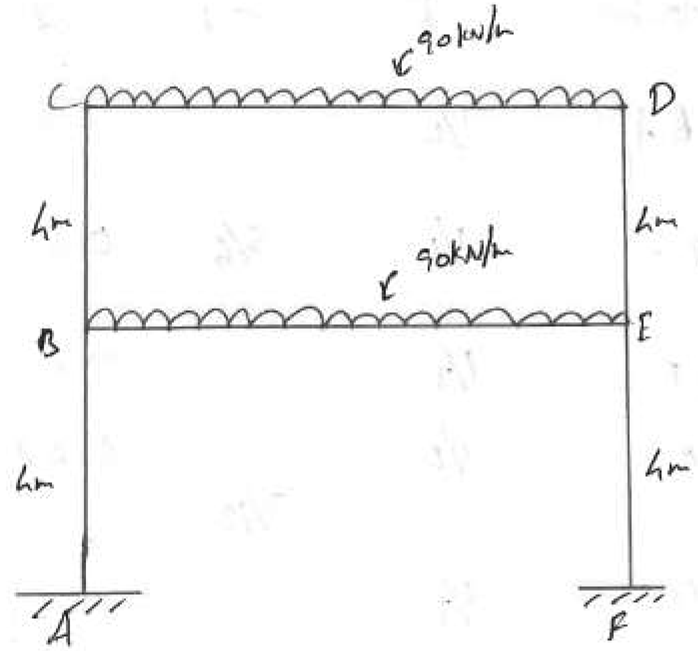
$M_{BC} = -39.981$ kN-m

$M_{CB} = 51.12$ kN-m

$M_{CD} = -49.704$ kN-m

$M_{DC} = 5.148$ kN-m

4. Analyze the beam shown in fig 4 using Kani's method & draw BMD.



Step 1: FEM

$$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCD} = M_{FDE} = M_{FED} = M_{FEF} = 0$$

$$M_{FBE} = \frac{-wl^2}{12} = \frac{-90\times6^2}{12} = -270 \text{ kN-m}$$

$$M_{FEB} = \frac{-wl^2}{12} = \frac{90\times6^2}{12} = 270 \text{ kN-m}$$

$$M_{FCD} = \frac{-wl^2}{12} = \frac{-90\times6^2}{12} = -270 \text{ kN-m}$$

$$M_{FDC} = \frac{+wl^2}{12} = \frac{90\times6^2}{12} = 270 \text{ kN-m}$$

Step 2: Relative Stiffness

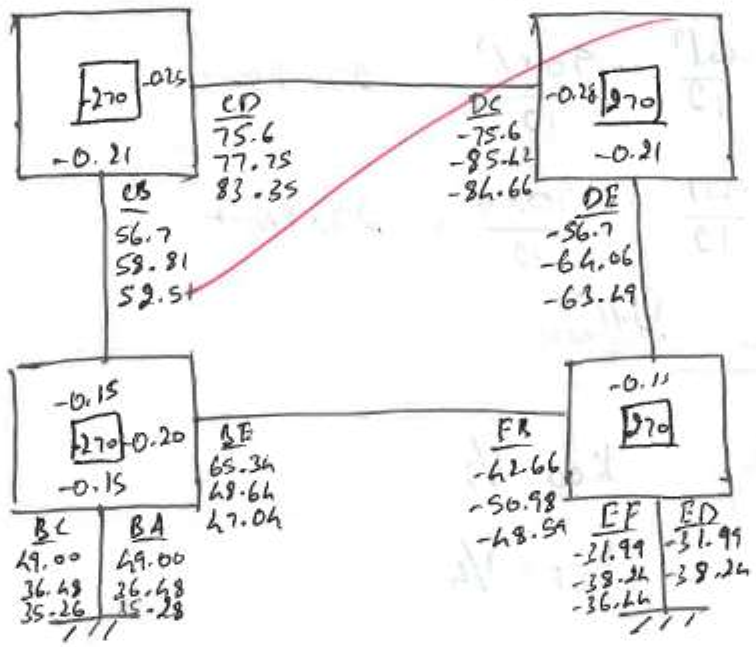
$K_{AB} = 1/4$ $K_{CD} = 1/3$

$K_{BE} = 1/3$ $K_{EF} = 1/4$

Step 3: Rotation Factor

Joint	Member	K	ΣK	d	R.F
B	BA	1/4		0.3	-0.15
	BC	1/4	5/6	0.3	-0.15
	BE	1/3		0.4	-0.20
C	CB	1/4	7/12	0.42	-0.21
	CD	1/3		0.57	-0.28
	DC	1/3		0.57	-0.28
D	DE	1/4	7/12	0.42	-0.21
	ED	1/4		0.3	-0.15
E	EF	1/4	5/6	0.3	-0.15
	EB	1/3		0.4	-0.20

Step 4:



Final Iteration

AB = 0

BA = 35.28   DE = -63.49   FE = 0

BC = 35.28   ED = -36.44

CB = 62.51   EF = -36.44

CD = 83.35   BE = 47.04

DC = -84.66   EB = -48.59

Step 5: Final Moments

$M_{AB} = 0 + 2(0) + 35.28 = 35.28$ kN-m

$M_{BA} = 0 + 2(35.28) + 0 = 70.56$ KN-m

$M_{BC} = 0 + 2(35.28) + 62.51 = 133.07$ KN-m

$M_{CB} = 0 + 2(62.51) + 35.28 = 160.3$ KN-m

$M_{CD} = -270 + 2(83.35) + (-84.66) = -187.96$ KN-m

$M_{DC} = 270 + 2(-84.66) + 83.35 = 184.03$ KN-m

$M_{DE} = 0 + 2(-63.49) + (-36.44) = -163.42$ KN-m

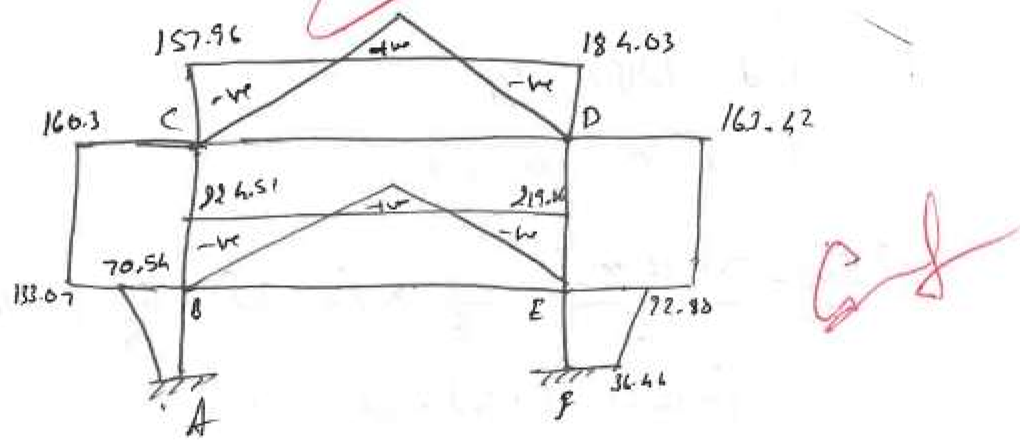
$M_{ED} = 0 + 2(-36.44) + (-63.49) = -136.37$ KN-m

$M_{EB} = 270 + 2(-48.59) + (47.04) = 219.86$ KN-m

$M_{BE} = -270 + 2(-47.04) + (-48.59) = -224.51$ KN-m

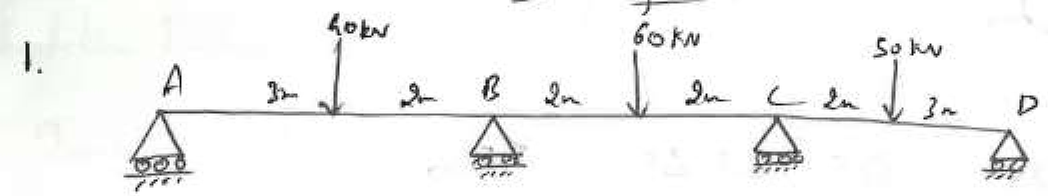
$M_{EF} = 0 + 2(-36.44) + 0 = -72.88$ KN-m

$M_{FE} = 0 + 2(0) + (-36.44) = -36.44$ KN-m



TD Assignment

1.



S-1: $D_s = 8-3 = 5$

S-2: $\Sigma M_A = 0$

$(40 \times 3) + (40 \times 7) + (50 \times 12) - (V_D \times 14) = 0$

$V_D = 77.86$ kN

$\Sigma M_D = 0$

$(V_A \times 14) - (40 \times 11) - (60 \times 7) - (50 \times 3) = 0$

$V_A = 72.14$ kN

S-3:

$$EI\frac{d^2y}{dx^2} = 72.14x - 40(x-3) - 60(x-7) - 50(x-11)$$

$$EI\frac{dy}{dx} = \frac{72.14x^2}{2} - \frac{40}{2}(x-3)^2 - \frac{60}{2}(x-7)^2 - \frac{50}{2}(x-11)^2 + C_1$$

$$EIy = \frac{72.14x^3}{6} - \frac{40}{6}(x-3)^3 - \frac{60}{6}(x-7)^3 - \frac{50}{6}(x-11)^3 + C_1x + C$$

at $x=0$, $y=0$ | at $x=14$, $y=0$

$C_2 = 0$ | $C_1 = -1461.69$

To find $D\varphi L_1 = y_B$

at $x=5$, $y_B = ?$

$$EIy_B = \frac{72.14 \times 5^3}{6} - \frac{40}{6}\times(5-3)^3 - \frac{60}{6}(5-7)^3 - \frac{50}{6}(5-11)^3 + (-1461.69 \times 5) + 0$$

$$D\varphi L_1 = \frac{-5858.85}{EI}$$

To find $D\varphi L_2 = y_C$

$$EIy_C = \frac{72.14}{6} \times 9^3 - \frac{40}{6}\times(9-3)^3 - \frac{60}{6}(9-11)^3 + (-1461.69 \times 9) + 0$$

$$D\varphi L_2 = \frac{-5910.2}{EI}$$

Apply unit load at B:

$\Sigma M_A = 0$

$(1 \times 5) - (R_D \times 14) = 0$

$R_D = 0.35$ KN

$\Sigma M_D = 0$

$R_A \times 14 + (1 \times 9) = 0$

$R_A = -0.64$ kN

Consider Fig (b)

$$EI\frac{dy}{dx} = -\frac{0.643x^2}{2} + \frac{(x-5)^2}{2} + C_1x + C_2$$

at $x=0$, $y=0$ | at $x=14$, $y=0$

$0 = 0 + 0 + C_1 \times 0 + C_2$ | $0 = -\frac{0.643}{6}(14)^3 + \frac{1}{6}(14-5)^3 + (1 \times 14)$

$\therefore C_2 = 0$ | $= -294.07 + 121.5 + 14 C_1$

$C_1 = 12.33$

To find $F_{11} = y_B$

at $x=5$, $y_B = ?$

$$EIy_B = -0.\frac{643}{6}(5)^3 + \frac{1}{6}(5-3)^3 + (12.33 \times 5) + 0$$

$$EIy_B = -13.4 + 0 + 61.63$$

$$F_{11} = \frac{48.25}{EI}$$

To find $F_{12} = y_C$

at $x=9$, $y_c = ?$

$$EIy_c = -\frac{0.643}{6}(9)^3 + \frac{(19-5)^3}{6} + (12.33 \times 9) + 0$$

$$EIy_c = -78.12 + 10.67 + 110.67$$

$$F_{21} = \frac{43.52}{EI}$$

Apply unit load @ c

$$\Sigma M_A = 0$$

$$(1 \times 9) - (R_D \times 14) = 0$$

$$R_D = 0.64 \text{ KN}$$

$$\Sigma M_D = 0$$

$$(R_A \times 14) - (1 \times 5) = 0$$

$$R_A = 0.36 \text{ KN}$$

Consider Fig (c)

$$EI\frac{d^2y}{dx^2} = 0.36(x) - 1(x-9)$$

$$EI\frac{dy}{dx} = -\frac{0.36}{2}x^2 - \frac{1}{2}(x-9)^2 + C_1$$

$$EI\frac{dy}{dx} = -\frac{0.36}{6}x^3 - \frac{1}{6}(x-9)^3 + C_1x + C_2$$

at $x=0$, $y=0$

$$C_2 = 0$$

at $x=14$, $y=0$

$$0 = -\frac{0.36}{6}(14)^3 + \frac{1}{6}(14-9)^3 + (C_1 \times 14) + 0$$

$$C_1 = 10.27$$

To find $F_{12} = y_B$

at Find $F_{12} = y_B$

at $x=5$, $y_B = ?$

$$EIy_B = -\frac{0.36(5)^3}{6} - \frac{1}{6}(5-9)^3 + (10.27 \times 5) + 0$$

$$EIy_B = -7.5 - 0 + 51.35$$

$$F_{12} = \frac{43.85}{EI}$$

To find $F_{22} = y_c$

at $x=9$, $y_c = ?$

$$EIy_c = \frac{0.36}{6}(9)^3 - \frac{1}{6}(9-9)^3 + (10.27 \times 9) + 0$$

$$EIy_c = -43.74 - 0 + 92.43$$

$$F_{22} = \frac{48.69}{EI}$$

Flexibility Matrix

$$F = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 48.25 & 43.85 \\ 43.52 & 48.69 \end{bmatrix}$$

$$[Q] = [F]^{-1}[DQL]$$

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = -\frac{1}{EI}\begin{bmatrix} 48.25 & 43.85 \\ 43.52 & 48.69 \end{bmatrix}^{-1}\begin{bmatrix} -5858.88 \\ -5910.2 \end{bmatrix}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \frac{-EI}{440.94}\begin{bmatrix} 48.69 & -43.25 \\ -43.26 & 48.25 \end{bmatrix}\begin{bmatrix} -5858.88/EI \\ -5910.2/EI \end{bmatrix}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 646.96 - 52.775 \\ -578.26 \quad 646.73 \end{bmatrix}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 59.21 \\ 68.47 \end{bmatrix}$$

$V_B = 59.21 \qquad V_c = 68.47$

$\Sigma M_A = 0$

$(40 \times 3) - (59.21 \times 5) + (60 \times 7) - (68.47 \times 9) + (50 \times 1)$

$- (R_D \times 14) = 0$

$V_D = 12.69$ KN

$\Sigma M_D = 0$

$(R_D \times 14) - (40 \times 11) + (59.21 \times 9) - (60 \times 7) + (68.47 \times 5)$

$- (50 \times 3) = 0$

$V_A = 9.63$ KN

$V_A + V_B + V_c + V_D = 40 + 60 + 50$

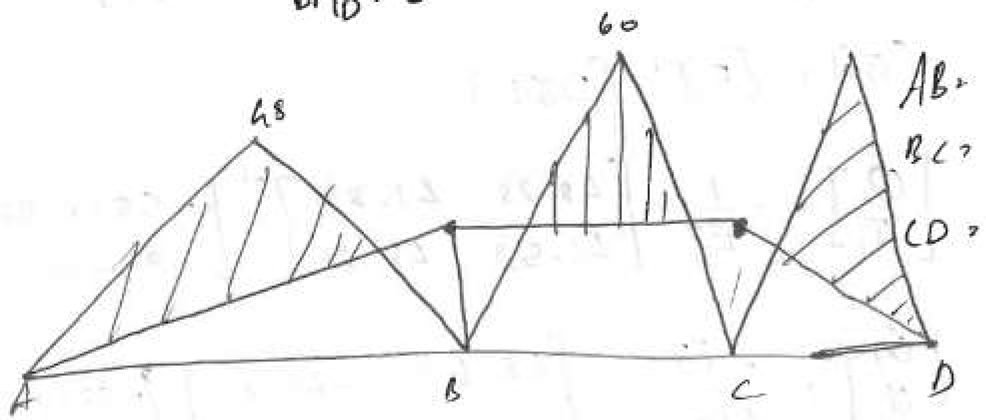
$150 = 150$

$BM_A = 0$

$BM_B = (9.63 \times 5) - (40 \times 2) = -31.25$

$BM_c = (12.69 \times 5) - (50 \times 2) = -36.55$

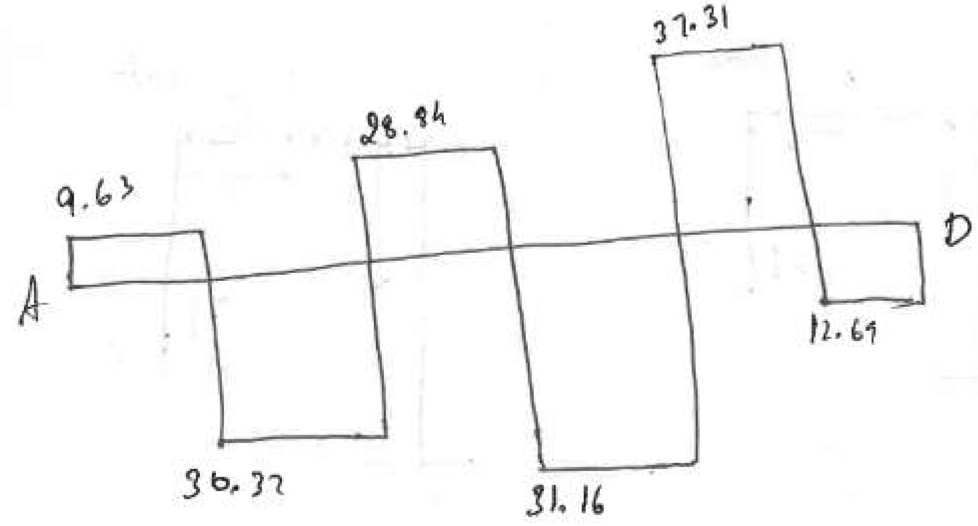
$BM_D = 0$



$AB = \frac{40 \times 3 \times 2}{5} = 48$

$BC = \frac{60 \times \cancel{16}}{\cancel{16}} = 60$

$CD = \frac{50 \times 2 \times 3}{5} = 60$



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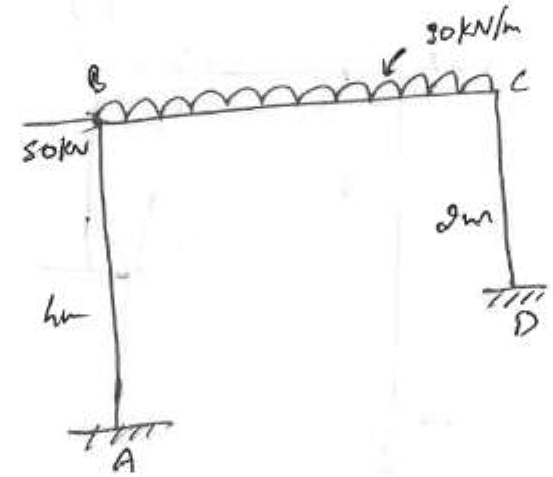
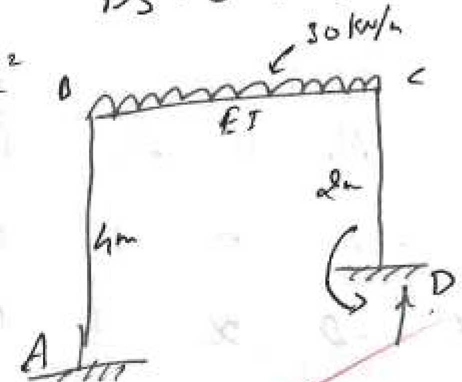
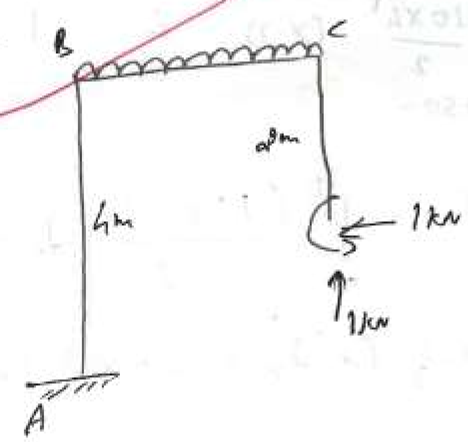


S-1 $\quad D_s = 6 - 3 = 3$

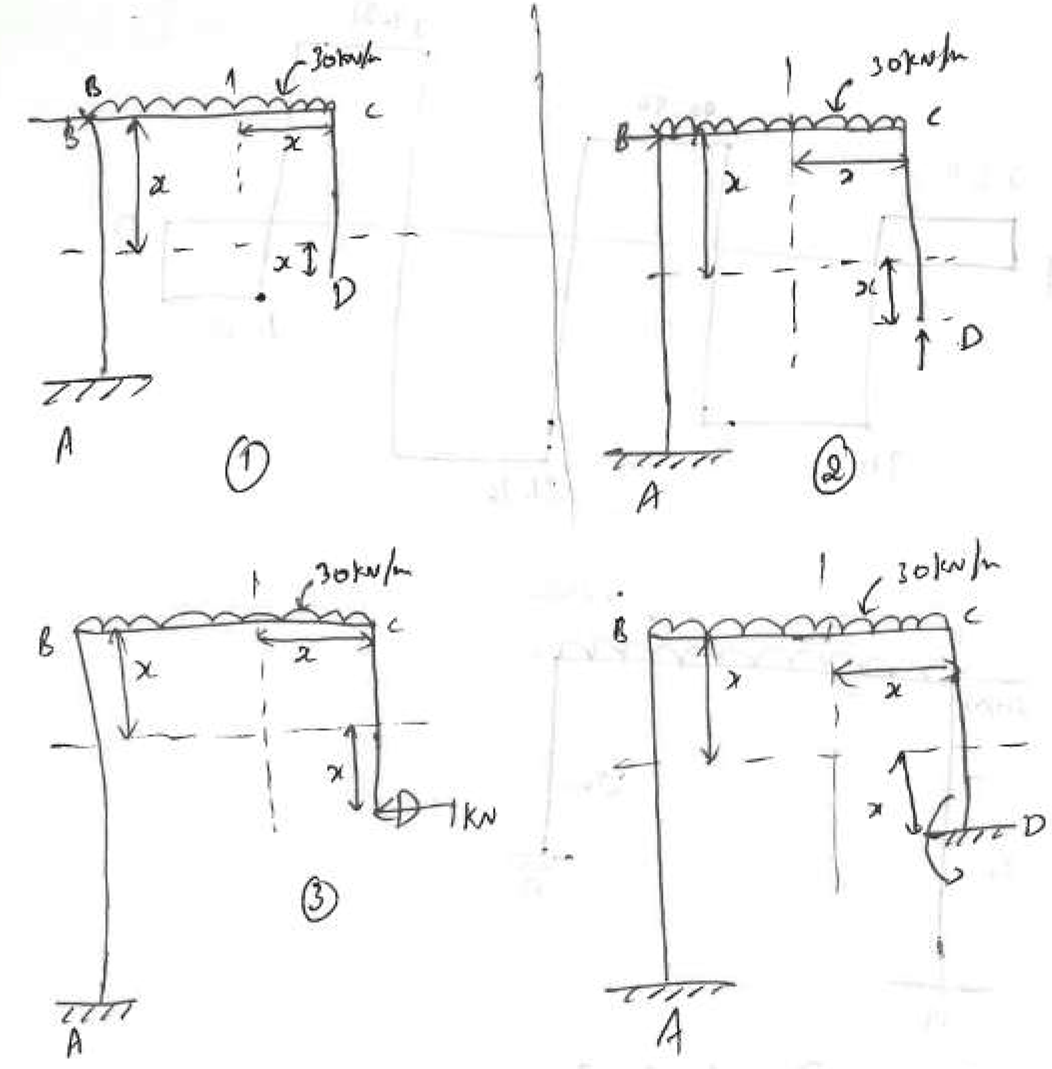
S-2



S-3



S-4?



S-5

Section	φ	M	M_1	M_2	M_3	limit
DC	D	0	$-1\times x$	0	1	0-2
CB	C	$\frac{-30x^2}{2}$	-2	x	1	0-4
BA	B	$\frac{-30\times 4^2}{2}+50x$	$(x-2)$	4	1	0-4

$$D\varphi_{c1} = \int_0^2 \frac{0\,x}{EI}dx + \int_0^4 \frac{(-15x^2)(2)}{EI}dx + \int_0^4 \frac{-(240+50x)(2-x)}{EI}dx$$

$$= \frac{1}{EI}\left[20\times\frac{1}{3}(x^3)_0^4 + 140\times\frac{1}{2}(x^2)_0^4 - 480\times(x)_0^4\right]$$

$$= \frac{1}{EI}\left[20\times\frac{1}{3}(4^3-0) + 70(4^2-0) - 480\times(4-0)\right]$$

$$= \frac{-373.34}{EI}$$

$$D\varphi_{2} = \int_0^2 \frac{0\times 0}{EI}dx + \int_0^4 \frac{(-15x^2)x}{EI}dx + \int_0^4 \frac{(240+350x)}{EI}\times 4\,dx$$

$$= -\frac{1}{EI}\left[15\times\frac{(x^4)_0^4}{4} + 200\times\frac{1}{2}(x^2)_0^4 + 960\times(x)_0^4\right]$$

$$= -\frac{1}{EI}\left[15\times\frac{1}{4}(4^4-0) + 100(4^2-0) + 960(4-0)\right]$$

$$= -\frac{6400}{EI}$$

$$D\varphi_{3} = \int_0^2 \frac{0(-1)}{EI}dx + \int_0^4 \frac{(-15x^2)(-1)}{EI}dx + \int_0^4 \frac{[-(240+50x)(-1)]}{EI}dx$$

$$= \frac{1}{EI}\left[15\times\frac{1}{3}(x^3)_0^4 + 240\times(x)_0^4 + 50\times\frac{1}{2}(x^2)_0^4\right]$$

$$= \frac{1}{EI}\left[15\times\frac{1}{3}(4^3) + 240\times 4 + 25(4^2)\right] = \frac{1680}{EI}$$

$$F_{11} = \int \frac{M_1^2}{EI}dx = \int_0^2 \frac{x^2}{EI}dx + \int_0^4 \frac{2\times 2}{EI}dx + \int_0^4 \frac{(2-x)(2-x)}{EI}dx$$

$$= \int_0^2 \frac{x^2}{EI}dx + \int_0^4 \frac{4}{EI}dx + \int_0^4 \frac{4x^2-4x+4}{EI}dx$$

$$= \int_0^2 x^2\frac{dx}{EI} + \int_0^4 \frac{x^2-4x+8}{EI}dx$$

$$= \frac{1}{EI}\left[\frac{1}{3}(x^3)_0^2 + \frac{1}{3}(x^3)_0^4 - \frac{4}{2}(x^2)_0^4 + 8(x)_0^4\right]$$

$$F_{11} = \frac{1}{EI}\left[\frac{1}{3}(x^3)_0^2 + \frac{1}{3}(x^3)_0^4 - \frac{4}{2}(x^2)_0^4 + 8(x)_0^4\right]$$

$$F_{11} = \frac{1}{EI}\left[\frac{1}{3}(8) + \frac{1}{3}4^3 - 2(4^2) + 8(4)\right] = \frac{24}{EI}$$

$$F_{22} = \int \frac{M_2^2}{EI}dx = \int_0^2 \frac{0}{EI}dx + \int_0^4 \frac{x^2}{EI}dx + \int_0^4 \frac{4^2}{EI}dx$$

$$= \frac{1}{EI}\left[\frac{1}{3}(x^3)_0^4 + 16(x)_0^4\right]$$

$$= \frac{1}{EI}\left[\frac{1}{3}(4^3) + 16(4)\right] = \frac{85.33}{EI}$$

$$F_{33} = \int \frac{M_3^2}{EI}\,dx = \int_0^2 \frac{(-1)^2}{EI}dx + \int_0^4 \frac{(-1)^2}{EI}dx + \int_0^4 \frac{(-1)^2}{EI}dx$$

$$= \frac{1}{EI}\left[(x)_0^2 + 2(x)_0^4\right] = \frac{1}{EI}\left[(2-0)+2(4)\right] = \frac{1}{EI}(2+8)$$

$$= \frac{10}{EI}$$

$$F_{12} = F_{21} = \int \frac{M_1 M_2}{EI}\,dx = \int_0^2 \frac{x(0)}{EI}dx + \int_0^4 \frac{2x}{EI}dx + \int_0^4 \frac{(2-x)4}{EI}dx$$

$$= \int_0^4 \frac{2x}{EI}dx + \int_0^4 \frac{8-4x}{EI}dx$$

$$= \frac{1}{EI}\left[-1(4^2) + 8(4)\right] = \frac{16}{EI}$$

$$F_{13} = F_{31} = \int \frac{M_1 M_3}{EI}\,dx = \int_0^2 \frac{x(-1)}{EI}dx + \int_0^4 \frac{2(-1)}{EI}dx + \int_0^4 \frac{(2-x)(-1)}{EI}dx$$

$$= \frac{1}{EI}\left[-\frac{1}{2}(2^2-0) + \frac{1}{2}(4)^2 - 4(4)\right]$$

$$= \frac{-10}{EI}$$

$$F_{23} = \int \frac{M_2 M_3}{EI}\,dx = \int_0^2 \frac{(0\times -1)}{EI}dx + \int_0^4 \frac{x(-1)}{EI}dx + \int_0^4 \frac{4(-1)}{EI}dx$$

$$= \frac{1}{EI}\left[-\frac{1}{2}(4)^2 - 4(4)\right] = \frac{-24}{EI}$$

$$\therefore F = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 24 & 16 & -10 \\ 16 & 85.33 & -24 \\ -10 & -24 & 10 \end{bmatrix}$$

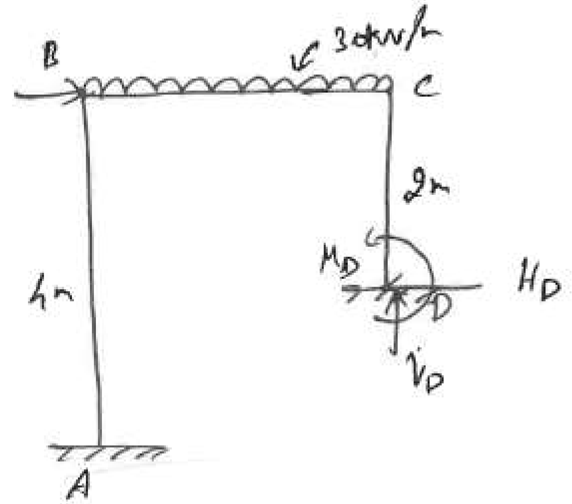
$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} = -\begin{bmatrix} 24 & 16 & -10 \\ 16 & 85.37 & -24 \\ -10 & -24 & 10 \end{bmatrix}^{-1} \begin{bmatrix} -373.34 \\ -6400 \\ 1630 \end{bmatrix}$$

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} -53.32 \\ 70.01 \\ -53.3 \end{bmatrix} = \begin{bmatrix} H_D \\ V_D \\ M_D \end{bmatrix}$$

$\therefore$ $H_D = -53.32$ kN

$V_D = 70.01$ kN

$M_D = -53.3$ kN



$$M_B = M_D + V_D\times 4 - H_D\times 2 - 30\times 4\times \frac{4}{2}$$

$$= 53.3 + (70.01\times 4) - (53.32\times 2) - 240$$

$$= -13.3 \text{ kN-m}$$

$$M_A = M_D - (50\times 4) - (30\times 4\times \tfrac{4}{2}) + (V_D\times 4) + (H_D\times 2)$$

$$= 53.3 - 200 - 240 + (70.01\times 4) + (53.32\times 2)$$

$M_A = 0$

$\therefore$ The Moments are

$M_{AB} = 0$

$M_{BA} = 13.3$

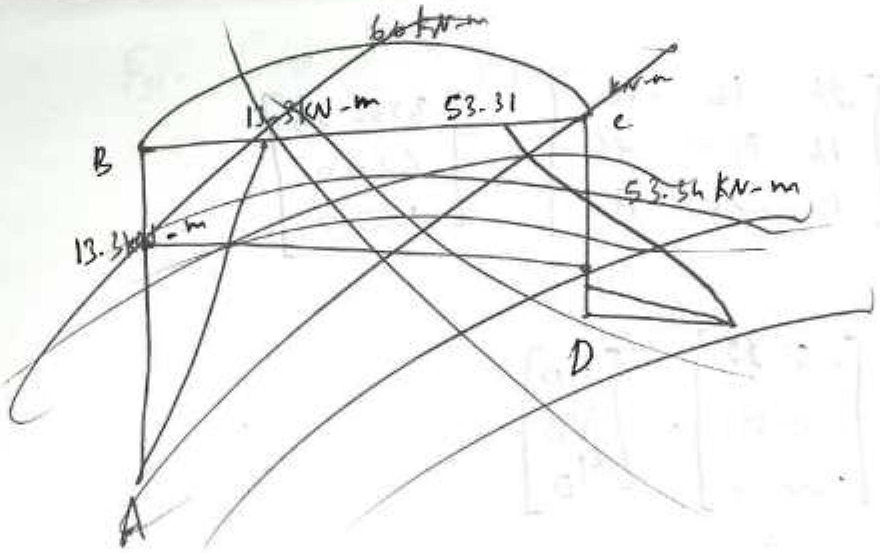
$M_{BC} = -13.3$ kN-m

$M_{CB} = 53.34$ kN-m

$M_{CD} = -53.34$ kN-m

$M_{DC} = -53.30$ kN-m

$$M = \frac{WL^2}{8} = \frac{30\times 4^2}{8} = 60 \text{ kN-m}$$


60 kN-m
13.3 kN-m
53.31
C
B
53.34 kN-m
13.3 kN-m
D
A

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Date
Page 1

ASSIGNMENT - 01

MODULE 04: STATISTICAL METHODS &

MODULE 01: CALCULUS OF COMPLEX FUNC^{N}'S

* Module 04: Statistical methods and curve fitting

1) Fit a straight line by the method of least square to each of the following data.

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

Sol: Let the eq^n of straight line be,

$\boxed{y = ax + b}$

normal equations are,

$$\boxed{\begin{aligned} \Sigma y &= a\Sigma x + n b \\ \Sigma xy &= a\Sigma x^2 + b\Sigma x \end{aligned}}$$

x	y	x^2	xy
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2

$n = 5$

$\Sigma x = 10$

$\Sigma y = 16.9$

$\Sigma x^2 = 30$

$\Sigma xy = 47.1$

$16.9 = a\,10 + b\,5$ $\quad a = 1.33$

$47.1 = a\,30 + b\,10$ $\quad b = 0.72$

$\boxed{y = 1.33x + 0.72}$ is the required eq^n which fits the data given.

2) Find the equation of the best fitting straight line in the following data & hence estimate the value of the dependent variable corresponding to the value 30 of the independent variable

x	5	10	15	20	25
y	16	19	23	26	30

sol: Let the eq^n of straight line be,

$y = ax + b$

normal equations are,

$\Sigma y = a\Sigma x + nb$

$\Sigma xy = a\Sigma x^2 + b\Sigma x$

x	y	x^2	xy
5	16	25	80
10	19	100	190
15	23	225	345
20	26	400	520
25	30	625	750

$n = 5$
$\Sigma x = 75$
$\Sigma y = 114$
$\Sigma x^2 = 1375$
$\Sigma xy = 1885$

$114 = a75 + b5$ $\quad a = 0.7$

$1885 = a1375 + b75$ $\quad b = 12.3$

$\boxed{y = 0.7x + 12.3}$ is the best fitting eq^n for the given data

when $x = 30$, $y = 0.7(30) + 12.3$

$\boxed{y = 33.3}$

3) A simply supported beam carries a concentrated load P at its mid point corresponding to various values of P the maximum deflection Y is measured & is given in the following table

P	100	120	140	160	180	200
Y	0.45	0.55	0.6	0.7	0.8	0.85

Find a law of the form $Y = a + bP$ & hence estimate Y when P is 150

sol: The given eq^n is $\boxed{Y = a + bP}$

normal equations of the given law,

$\Sigma Y = an + b\Sigma P$

$\Sigma PY = a\Sigma P + b\Sigma P^2$

P	Y	P^2	PY
100	0.45	10000	45
120	0.55	14400	66
140	0.6	19600	84
160	0.7	25600	112
180	0.8	32400	144
200	0.85	40000	170

$n = 6$
$\Sigma Y = 3.95$
$\Sigma P^2 = 142000$
$\Sigma PY = 621$
$\Sigma P = 900$

$3.95 = a6 + b900$ $\quad a =$

$621 = a900 + b142000$ $\quad b =$

$a = 0.0476$; $b = 0.00407$

$\boxed{Y = 0.0476 + 0.0041P}$ is the eq^n for the maximum deflection of the Y for given data. when $P = 150$, $Y = 0.0476 + 0.0041(150)$

$\boxed{Y = 0.6626}$

4) Fit a second degree parabola of the form $y = ax^2 + bx + c$ to the following data

x	0	1	2	3	4	5
y	1	3	7	13	21	31

sol: given eq^n of parabola, $y = ax^2 + bx + c$

normal eq^ns are, $\Sigma y = a\Sigma x^2 + b\Sigma x + nc$

$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$

$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$

x	y	xy	x^2	x^3	x^4	x^2y
0	1	0	0	0	0	0
1	3	3	1	1	1	3
2	7	14	4	8	16	28
3	13	39	9	27	81	117

$\Sigma x = 15$; $\Sigma y = 76$; $\Sigma x^2 = 55$, $\Sigma xy = 295$
$\Sigma x^3 = 225$ $\Sigma x^4 = 979$, $\Sigma x^2 y = 1259$

$76 = a\,55 + b\,15 + c\,6$
$295 = a\,225 + b\,55 + c\,15$
$1259 = a\,979 + b\,225 + c\,55$
$a = 1$, $b = 1$, $c = 1$
$\therefore$ $\boxed{y = x^2 + x + 1}$ is the eq^n that best fits the given data.

5) Fit a second degree parabola $y = ax^2 + bx + c$ in the least square to the following data & hence estimate y at $x = 6$

x	1	2	3	4	5
y	10	12	13	16	19

$\therefore$ given eq^n of parabola is $y = ax^2 + bx + c$
normal eq^{n}s are: $\Sigma y = a\Sigma x^2 + b\Sigma x + nc$
$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$
$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$

x	y	xy	x^2	x^2y	x^3	x^4
1	10	10	1	10	1	1
2	12	24	4	48	8	16
3	13	39	9	117	27	81
4	16	64	16	256	64	256
5	19	95	25	475	125	625

$\Sigma x = 15$; $\Sigma xy = 232$; $\Sigma x^3 = 225$
$\Sigma y = 70$; $\Sigma x^2 y = 906$ $\Sigma x^4 = 979$
$\Sigma x^2 = 55$
$n = 5$

$70 = a\,55 + b\,15 + c\,5$
$232 = a\,225 + b\,55 + c\,15$
$906 = a\,979 + b\,225 + c\,55$
$a = 0.2857$; $b = 0.4857$, $c = 9.4$
$\boxed{\therefore y = 0.2857x^2 + 0.4857x + 9.4}$ is the eq^n that best fits the curve.
when $x = 6$, $y = 0.2857(6)^2 + 0.4857(6) + 9.4$
$\boxed{y = 22.5994}$

6) Fit a second degree parabola of the form $y = a + bx + cx^2$ to the following data

x	-2	-1	0	1	2
y	-3.15	-1.39	0.62	2.88	5.378

Sol: the given form is $y = a + bx + cx^2$
normal eq^n are $\Sigma y = an + b\Sigma x + c\Sigma x^2$
$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$
$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

x	y	xy	x^2	x^3	x^4	x^2y
-2	-3.15	6.3	4	-8	16	-12.6
-1	-1.39	1.39	1	-1	1	-1.39
0	0.62	0	0	0	0	0
1	2.88	2.88	1	1	1	2.88
2	5.378	10.756	4	8	16	21.512

$\Sigma x = 0$; $\Sigma y = 4.338$; $\Sigma xy = 21.326$; $\Sigma x^2 = 10$
$\Sigma x^3 = 0$ $\Sigma x^4 = 34$; $\Sigma x^2 y = 10.402$, $n = 5$
$4.338 = a\,5 + b\,0 + c\,10$
$21.326 = a\,0 + b\,10 + c\,0$
$10.402 = a\,10 + b\,0 + c\,34$
$a = 0.6210$; $b = 2.1326$; $c = 0.1233$
$\boxed{\therefore y = 0.6210 + 2.1326x + 0.1233x^2}$ is the

7) In a partially destroyed lab record, only the lines of regression of y on x & x on y available as $4x-5y+33=0$ & $20x-9y=107$ respectively. Calculate $\bar{x}$, $\bar{y}$ & coefficient of correlation b/w x & y

Sol: We know that the regression lines of the records passes through $(\bar{x},\bar{y})$

$4\bar{x}-5\bar{y}+33=0 \Rightarrow 4\bar{x}-5\bar{y}=-33$

$20\bar{x}-9\bar{y}=107 \Rightarrow 20\bar{x}-9\bar{y}=107$

$\bar{x}=13$, $\bar{y}=17$

$\boxed{\therefore \bar{x}=13 \ \& \ \bar{y}=17}$

w.k.t coefficient of correlation,

$\boxed{r=\sqrt{(\text{coeff of } x)\cdot(\text{coeff } y)}}$

$r=$

$4x-5y=-33$ (y on x)

$-5y=-33-4x$

$\boxed{y=\frac{4}{5}x+\frac{33}{5}}$

$20x-9y=107$ (x on y)

$20x=107+9y$

$\boxed{x=\frac{9}{20}y+\frac{107}{20}}$

$\therefore r=\sqrt{(4/5)\times(9/20)}$

$\boxed{r=0.6}$ $\therefore$ coefficient of correlation

(8) Obtain the coefficient correlation for the following data

x	10	14	18	22	26	30
y	18	12	24	6	30	36

Sol: Let coefficient correlation $\boxed{r=\frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}}}$

$X=x-\bar{x}$; $Y=y-\bar{y}$

$\bar{x}=\frac{\Sigma x}{n}=\frac{120}{6}=20$

$\bar{y}=\frac{\Sigma y}{n}=\frac{126}{6}=21$

x	y	X	Y	X^2	Y^2	XY
10	18	-10	-3	100	9	30
14	12	-6	-9	36	81	54
18	24	-2	3	4	9	-6
22	6	2	-15	4	225	-30
26	30	6	9	36	81	54
30	36	10	15	100	225	150

$\Sigma x=120$, $\Sigma y=21$, $\Sigma X=0$; $\Sigma Y=0$

$\Sigma XY=252$ $\Sigma X^2=~~1504~~\ 280$, $\Sigma Y^2=630$

$r=\frac{252}{\sqrt{280}\sqrt{630}}=~~0.2589~~$

$\boxed{\therefore r=0.6}$ is positively correlated

9) Calculate the coefficient of correlation & obtain the line of regression of the following data

x	1	2	3	4	5	6	7	8
y	9	8	10	12	11	13	14	16

also compute y correspond to x = 6.2.

Sol: Coefficient of correlation $\boxed{r=\frac{\sigma_x^2+\sigma_y^2-\sigma_z^2}{2\sigma_x\sigma_y}}$

where $z=x-y$.

x	y	z	x^2	y^2	z^2
1	9	-8	1	81	64
2	8	-6	4	64	36
3	10	-7	9	100	49
4	12	-8	16	144	64
5	11	-6	25	121	36
6	13	-7	36	169	49
7	14	-7	49	196	49
8	16	-8	64	256	64

$\boxed{\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2} = \frac{204}{8} - (4.5)^2 \Rightarrow \boxed{\sigma_x^2 = 5.25 \quad \sigma_x = 2.2913}$

$\boxed{\bar{x} = \frac{\Sigma x}{n}} = \frac{36}{8} = 4.5, \boxed{\bar{x} = 4.5}$

$\boxed{\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2} = \frac{1131}{8} - (11.625)^2 \Rightarrow \boxed{\sigma_y^2 = 6.2344 \quad \sigma_y = 2.4969}$

$\boxed{\bar{y} = \frac{\Sigma y}{n}} = \frac{93}{8} = 11.625 \Rightarrow \boxed{\bar{y} = 11.625}$

$\boxed{\sigma_z^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2} = \frac{411}{8} - (-7.125)^2 = \boxed{\sigma_z^2 = 0.6094}$

$\boxed{\bar{z} = \frac{\Sigma z}{n}} = \frac{-57}{8} = -7.125 \Rightarrow \boxed{\bar{z} = -7.125}$

$\therefore r = \frac{5.25 + 6.2344 - 0.6094}{2 \times 2.2913 \times 2.4969} = 0.9504$

$\boxed{\therefore r = 0.9504}$ which is perfectly strongely correlated

→ Lines of regression; y on x

$\boxed{(y - \bar{y}) = r \frac{\sigma_y}{\sigma_x}(x - \bar{x})}$

$y - 11.625 = 0.9504 \left(\frac{2.4969}{3.0822}\right)(x - 4)$

$y = 0.79699x - 3.0797 + 11.625$

$\boxed{y = 0.7969x + 8.5453}$

x on y; $\boxed{x - \bar{x} = r \frac{\sigma_x}{\sigma_y}(y - \bar{y})}$

$x - 4 = 0.9504 \left(\frac{3.0822}{2.4969}\right)(y - 11.625)$

$x = 1.1732y - 13.6383 + 4$

$\boxed{x = 1.1732y - 9.6383}$

$\therefore$ when $x = 62$,

$y = 0.7969(62) + 8.4406$

$y = 56.1744$ $\therefore$ when $x = 62$, $\boxed{y = 56.1744}$

10) Find the correlation coefficient & the regression lines for the following data

x	1	2	3	4	5
y	2	5	3	8	7

Sol:

x	y	z	x^2	y^2	z^2
1	2	-1	1	4	1
2	5	-3	4	25	9
3	3	0	9	9	0
4	8	-4	16	64	16
5	7	-2	25	49	4

$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = \frac{55}{5} - (3)^2 \Rightarrow \boxed{\sigma_x^2 = 2 \quad \sigma_x = 1.4142}$

$\bar{x} = \frac{\Sigma x}{n} = \frac{15}{5} = 3 \Rightarrow \boxed{\bar{x} = 3}$

$\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2 = \frac{151}{5} - (5)^2 = 5.2 \Rightarrow \boxed{\sigma_y^2 = 5.2 \quad \sigma_y = 2.2804}$

$\bar{y} = \frac{\Sigma y}{n} = \frac{25}{5} = 5 \Rightarrow \boxed{\bar{y} = 5}$

$$\sigma_z^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2 = \frac{30}{5} - (-2)^2 \Rightarrow \boxed{\sigma_z^2 = 2}$$

$$\bar{z} = \frac{\Sigma z}{n} = \frac{-10}{5} \Rightarrow \boxed{\bar{z} = -2}$$

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_z^2}{2\sigma_x \sigma_y} \quad (z = x - y)$$

$$r = \frac{2 + 5.2 - 2}{2 \times 1.4142 \times 2.2804} \Rightarrow \boxed{r = 0.8062}$$

∴ The coeffient of correlation is 0.8062

The lines of regression: y on x,

$$\boxed{y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})}$$

$$y - 5 = 0.8062\left(\frac{2.2804}{1.4142}\right)(x - 3)$$

$$y = 1.2999x - 3.8999 + 5$$

$$\boxed{y = 1.2999x + 1.1001}$$

x on y, $(x - \bar{x}) = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$$(x - 3) = 0.8062\left(\frac{1.4142}{2.2804}\right)(y - 5)$$

$$x = 0.4999y - 2.4998 + 3$$

$$\boxed{x = 0.4999y + 0.5}$$

11) Compute the rank correlation coeffinient for the following data

x	68	64	75	50	65	80	77	40	55	69
y	62	58	68	45	81	60	63	48	50	70

Sol: $\boxed{\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}}$

x	y	Rx	Ry	di	di²
68	62	5	5	0	0
64	58	7	7	0	0
75	68	3	3	0	0
50	45	9	10	-1	1
65	81	6	1	5	25
80	60	1	6	-5	25
77	63	2	4	-2	4
40	48	10	9	1	1
55	50	8	8	0	0
69	70	4	2	2	4

$\Sigma di^2 = 60$

$$\rho = 1 - \frac{6(60)}{10(10^2-1)}$$

$$= 1 - \frac{360}{990} \Rightarrow \boxed{\rho = 0.6364}$$

The rank correlation coeffinient is 0.6364 is positively correleated.

12) The value of Spearman's rank correlation coefficient for certain pair of numbers of observations, was found to be 2/3. The sum of squares of the difference b/w corresponding rank was 55. Find the number of pairs.

Sol: $\boxed{\rho = \frac{2}{3}}$, $di^2 = 55$, $n = ?$

$$\boxed{\rho = 1 - \frac{\Sigma 6d^2}{n(n^2-1)}} \Rightarrow \frac{2}{3} = 1 - \frac{6 \times 55}{n(n^2-1)}$$

$$\cancel{\frac{2}{3} = 1 - \frac{\Sigma 6 \times 55}{n(n^2-1)}} \qquad -1 + \frac{2}{3} = \frac{6 \times 55}{n(n^2-1)}$$

(or) ⇒ $n^3 - n = 990 \Rightarrow n^3 - n - 990 = 0$

$\boxed{n = 10}$

$\cancel{0.6667 = 1 - \frac{330}{n(n^3-1)}}$

$\cancel{n(n^3-1)\ 0.6667 = n(n^3-1) - 300}$

$\cancel{0.6667n^3 - 0.6667 = 300n^3 + 300}$

$\cancel{0.6667n^3 - 300n^3 = 300 + 0.6667}$

$\cancel{n^3(0 - 0.6667 - 300 = 300n^3 - 0.6667n^3}$

$\cancel{-300.6667 = n^3(299.3333)}$

$\cancel{n^3 = \frac{-300.6667}{299.3333}}$

$\cancel{n^3 = -1}$

$\cancel{\boxed{n = -1}}$

$\frac{1}{3} = \frac{6 \times 55}{n(n^2-1)}$

$\frac{1}{3} = \frac{330}{n(n^2-1)} \Rightarrow 990 = n(n^2-1)$

(or)

$n(n^2-1) = 10 \times 99$

$10(100-1) = 10 \times 99$

$10(10^2-1) = 10 \times 99$

$\therefore \boxed{n = 10}$ ∴ there were 10 students

13) Ten competitors in a beauty contest are ranked by 3 judges in the following order:

1st judge	1	6	5	10	3	2	4	9	7	8
2nd judge	3	5	8	4	7	10	2	1	6	9
3rd judge	6	4	9	8	1	2	3	10	5	7

Sol: To check which 2 judges decision is acceptable we will use Spearman rank correlation method.

$\rho_{I, II} = \rho_{xy}$

$\Sigma di^2 = 200$

$\rho_{I, III} = 1 - \left[\frac{6 \Sigma di^2}{n(n^2-1)}\right] = 1 - \left[\frac{6 \times 200}{10(10^2-1)}\right]$

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x	y	di = x-y	di²
1	3	-2	4
6	5	1	1
5	8	-3	9
10	4	6	36
3	7	-4	16
2	10	-8	64
4	2	2	4
9	1	8	64
7	6	1	1
8	9	-1	1

y	z	di	di²
3	6	-3	9
5	4	1	1
8	9	-1	1
4	8	-4	16
7	1	6	36
10	2	8	64
2	3	-1	1
1	10	-9	81
6	5	1	1
9	7	2	4

$\Sigma di^2 = 214 \Rightarrow \rho = 1 - \left[\frac{6 \Sigma di^2}{n(n^2-1)}\right]$

$\rho_{II, III} = 1 - \left[\frac{6 \times 214}{10(10^2-1)}\right] \Rightarrow \boxed{\rho_{II, III} = -0.297}$

x	z	di	di^2
1	6	-5	25
6	4	2	4
5	9	-4	16
10	8	2	4
3	1	2	4
2	2	0	0
4	3	1	1
9	10	-1	1
7	5	-2	4
8	7	1	1

$\Sigma di^2 = 60$

$\rho_{I,III} = 1 - \boxed{\dfrac{6 \times 60}{10(10^2-1)}}$; $\boxed{\rho_{I,III} = 0.6364}$

Decision of I, II & III, II are -ve both are moving in opposite direction, but III, I are positive & hence moving in same method.

Application of Correlation & regression

14) Quotations of index number of equity share prices of a certain joint stock company & of prices of preference shares as given below.

Years	1991	1992	1993	1994	1995	1996	1997
Equity shares	97.5	99.4	98.6	96.2	95.1	98.4	97.1
Preference shares	75.1	75.9	77.1	78.2	79.0	74.8	76.2

Use the method of rank correlation to determine the relationship b/w equity share & preference share prices.

Sol: Let equity shares be x & preference shares be y

x	y	Rx	Ry	di	di^2
97.5	75.1	4	6	-2	4
99.4	75.9	1	5	-4	16
98.6	77.1	2	3	-1	1
96.2	78.2	6	2	4	16
95.1	79.0	7	1	6	36
98.4	74.8	3	7	-4	16
97.1	76.2	5	4	1	1

$\rho = 1 - \dfrac{6\Sigma d^2}{n(n^2-1)}$ $\Rightarrow \rho = 1 - \dfrac{540}{336} = 1 - 1.6071$

$= 1 - \dfrac{6(90)}{7(7^2-1)}$ $\boxed{\rho = -0.6071}$

$\therefore$ x & y are -vely correlated with each other $\therefore$ equity shares & preference shares are negatively correlated to each other. and move away from eachother

15) Assuming that we conduct an experiment with eight fields planted with corn, four fields having no nitrogen fertilizer & 4 fields having 80kgs of nitrogen fertilizer. The resulting corn yields are shown in the table in bushels per acre.

i) Compute a linear regression eq^n by least squares. Explain the meaning of regression equation in terms of fertilizers

Eg Corn yield.

ii) Predict corn yield for a field treated with 60kgs of fertilizers.

field	1	2	3	4	5	6	7	8
Nitrogen (kgs)(x)	0	0	0	0	80	80	80	80
Corn yield (acre)(y)	120	360	60	180	1280	1120	1120	700

x	y	$X=x-\bar{x}$	$Y=y-\bar{y}$	XY	X^2	Y^2
0	120	-40	-497.5	19900	1600	247506.25
0	360	-40	-257.5	10300	1600	66306.25
0	60	-40	557.5	22300	1600	310806.25
0	180	-40	-437.5	17500	1600	191406.25
80	1280	40	662.5	26500	1600	438906.25
80	1120	40	502.5	20100	1600	252506.25
80	1120	40	502.5	20100	1600	252506.25
80	700	40	82.5	3300	1600	6806.25

$\bar{x} = \frac{\Sigma x}{n} = \frac{320}{8} = 40$

$\bar{y} = \frac{\Sigma y}{n} = \frac{4940}{8} = 617.5$

$\boxed{r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}}}$

$\Sigma XY = 140000$
$\Sigma X^2 = 12800$
$\Sigma Y^2 = 1766750$
$\Sigma x^2 = 25600$
$\Sigma y^2 = 4817200$

$\boxed{r = 0.9309}$

$\sigma x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = 1600 \Rightarrow \sigma x = 40$

$\sigma y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2 = 220843.75$

$\sigma y = 469.9402$

lines of regression : $\boxed{y - \bar{y} = r\frac{\sigma y}{\sigma x}(x - \bar{x})}$

$y - 617.5 = 0.9309\left(\frac{469.9402}{40}\right)(x - 40)$

$= 10.9367x - 437.468 + 617.5$

$\boxed{y = 10.9367x + 180.032}$

when $x = 60$kgs,

$y = 10.9367(60) + 180.032$

$\boxed{y = 836.234}$

$\therefore$ coefficient of variation $\boxed{r = 0.9309}$

lines of regressions : $\boxed{y = 10.9367x + 180}$

of Eg $\boxed{x = 0.0792y - 8.9279}$

$x - \bar{x} = r\left(\frac{\sigma x}{\sigma y}\right)(y - \bar{y})$

$x - 40 = 0.9309\left(\frac{40}{469.9402}\right)(y - 617.5)$

$x - 40 = 0.0792y - 48.899279$

$\boxed{x = 0.0792y - 8.9279}$

$r = \sqrt{(\text{coeff } x)(\text{coeff } y)} = 0.9307$

(verification)

$\therefore$ at 60 kgs of fertilizers, corn yield will be 836.234 acre for a field.

* MODULE 01 - COMPLEX ANALYSIS

1) Define analytic function & obtain Cauchy - Reimann equation in polar form

Sol: A complex valued function $W=f(z)$ is said to be analytic at the point $z=z_0$ if $dw/dz = f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z)-f(z)}{\delta z}$ exists & is unique at z_0 & in the neighbourhood of z_0. Further $f(z)$ is said to be analytic in a region if it is analytic at every point of region.

Let (r,θ) be the polar coordinates of a point whose cartesian coordinates are (x,y) then $Z = re^{i\theta}$ is the polar form. $f(z) = u+iv$

$$u+iv = f(re^{i\theta}) \longrightarrow ①$$

diff w.r.t r & θ,

$$\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r} = f'(re^{i\theta}) \cdot e^{i\theta}$$

$$\frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial r} = ir\left\{\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right\} \qquad \boxed{i^2=-1}$$

$$= ir\frac{\partial u}{\partial r} - r\frac{\partial v}{\partial r}$$

$\therefore$ equating real & imaginary parts,

$$\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r} \quad \& \quad \frac{\partial v}{\partial \theta} = r\frac{\partial u}{\partial r}$$

$$\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta} \quad \& \quad \frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$$

$\therefore$ CR eq^n in polar form,

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \quad \& \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

diff w.r.t r & θ,

$$\frac{\partial^2 u}{\partial r^2} = -\frac{1}{r^2}\frac{\partial v}{\partial \theta} + \frac{1}{r}\frac{\partial^2 v}{\partial \theta \partial r}$$

$$\frac{\partial^2 u}{\partial \theta^2} = -r\frac{\partial^2 v}{\partial r \partial \theta}$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = -\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{r}\frac{\partial^2 v}{\partial \theta \partial r} + \frac{1}{r}\left[\frac{1}{r}\frac{\partial v}{\partial \theta}\right] + \frac{1}{r^2}\left[-r\frac{\partial^2 v}{\partial r \partial \theta}\right]$$

$$= \cancel{-\frac{1}{r^2}\frac{\partial v}{\partial \theta}} + \cancel{\frac{1}{r}\frac{\partial^2 v}{\partial \theta \partial r}} + \cancel{\frac{1}{r^2}\frac{\partial v}{\partial \theta}} - \cancel{\frac{1}{r}\frac{\partial^2 v}{\partial r \partial \theta}}$$

$$= 0$$

$$\boxed{\therefore \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0}$$

hence CR eq^n in polar form.

2) Show that z^n is analytic & find its derivative

Sol: Considering, $f(z) = (re^{i\theta})^n \qquad z = re^{i\theta}$

$$= r^n \times e^{i\theta n}$$

$$= r^n[\cos n\theta + i\sin n\theta]$$

$$f(z) = r^n\cos n\theta + i\, r^n \sin n\theta$$

$$f(z) = u+iV \text{ (general form)}$$

$$\boxed{u = r^n\cos n\theta \quad \& \quad V = r^n \sin n\theta}$$

diff u & v w.r.t r & θ,

$$\frac{\partial u}{\partial r} = n r^{n-1}\cos n\theta \qquad \frac{\partial v}{\partial r} = n r^{n-1}\sin n\theta$$

$$\frac{\partial u}{\partial \theta} = -n r^n \sin n\theta \qquad \frac{\partial v}{\partial \theta} = n r^n \cos n\theta$$

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \rightarrow ① \quad \&$$

$$\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta} \rightarrow ②$$

$n r^{n-1} \cos n\theta = \frac{1}{r} n r^n \cos n\theta$

$n r^{n-1} \cos n\theta = n r^{n-1} \cos n\theta$

$\therefore$ ① is satisfied $\longrightarrow$ eq^n ②,

$n r^{n-1} \sin \theta n = \frac{-1}{r}(-n r^n \sin n\theta)$

$n r^{n-1} \sin n\theta = n r^{n-1} \sin \theta$

$\therefore$ ② is satisfied

$\therefore$ both CR eq^{n}s are satisfied and all partisfied and all partial derivatives are continous. $\therefore f(z)$ is analytic

3) Show that $\omega = f(z) = \log z$ $(z \neq 0)$ is analytic using Cauchy-Riemann equation

Sol: As we cannot write, $f(z) = \log(x+iy)$ as $\log(x+iy)$ cannot simplify.

$\therefore z = re^{i\theta}$, $f(z) = \log(re^{i\theta})$

$= \log r + \log e^{i\theta}$

$= \log r + i\theta \log e \rightarrow ①$

$= \log r + i\theta$

$\therefore f(z) = u + iv$ (general form)

$\boxed{u = \log r \ \& \ v = \theta}$

$\Rightarrow$ CR eq^{n}s in polar form,

diff partially w.r.t r & θ

$\frac{\partial u}{\partial r} = \frac{1}{r}$; $\frac{\partial u}{\partial \theta} = 0$

$\frac{\partial v}{\partial r} = 0$; $\frac{\partial v}{\partial \theta} = 1$

$\boxed{\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \qquad \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}}$

$\frac{1}{r} = \frac{1}{r}(1) \qquad 0 = \frac{-1}{r}(0)$

$\therefore$ both the CR eq^{n}s are satisfied

$\therefore$ the partial derivatives are continuous.

$\therefore f(z)$ is analytic

4) ~~Find the analytic function $f(z) = u+iv$, given $u+v = x+y+e^x(\cos y - \sin y)$~~

~~Sol:~~

5) Verify for the fun^n $f(z) = \sinh z$ is analytic function or not & hence find its derivative.

Sol: By data, $f(z) = \sinh z$

$\boxed{\sinh x = -i\sin(ix)}$

$u + iv = -i \sin(iz)$

$= -i \sin\{i(x+iy)\}$

$= -i \sin(ix - y)$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$= i\{\sin ix \cdot \cos y - \cos ix \cdot \sin y\}$

$= (-i)\{i \sinh x \cos y - \cos ix \sin y\}$

$= (-i)\{i. \sinh x \cos y - \cosh x \sin y\}$

$u + iv = \sinh x \cos y + i \cosh x \sin y$

$\Rightarrow u = \sinh x \cos y$; $v = \cosh x \sin y$

$\frac{\partial u}{\partial x} = \cosh x \cos y \quad \frac{\partial v}{\partial x} = \sinh x \sin y$

$\frac{\partial u}{\partial y} = -\sinh x \sin y$; $\frac{\partial v}{\partial y} = \cosh x \cos y$

CR eq^n $u_x = v_y$ & $v_x = -u_y$ are satisfied & the partial derivations u_x, u_y, v_x & v_y are continuous

$\therefore f(z)$ is analytic

To find $f'(z)$ we have $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

$f'(z) = \cosh x \cos y + i \sinh x \sin y$

put $x = z$, $y = 0$,

$f'(z) = \cosh z \times 1 + i \sinh z \times 0$

6) Prove that real & imaginary part of an analytic funⁿ are harmonic.

Sol: A function is said to be harmonic if

$$\boxed{\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0}$$

i) Cartesian form: $f(z)=u+iv$ is an analytic

Let $f(z)=u(x,y)+iv(x,y)$ be analytic we shall show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$$

Since $f(z)$ is analytic,

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}\rightarrow(1) \quad \& \quad \frac{\partial v}{\partial x}=-\frac{\partial u}{\partial y}\rightarrow(2)$$

diff (1) wrt x & eqⁿ (2) wrt y partially

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 v}{\partial y\partial x} \quad \& \quad \frac{\partial^2 v}{\partial x\partial y}=-\frac{\partial^2 v}{\partial y^2}$$

equating v & u terms,

$$\frac{\partial^2 v}{\partial y\partial x}=\frac{\partial^2 v}{\partial x\partial y}\ ;\ \frac{\partial^2 u}{\partial x^2}=-\frac{\partial^2 u}{\partial y^2}$$

$$\therefore \boxed{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0}$$

$\therefore u$ is harmonic which is the real part of $f(z)$

Now, diff (1) wrt y & (2) wrt x,

$$\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 v}{\partial y^2} \quad \& \quad \frac{\partial^2 v}{\partial x^2}=-\frac{\partial^2 v}{\partial y\partial x}$$

$$\frac{\partial^2 v}{\partial y^2}=-\frac{\partial^2 v}{\partial x^2} \quad \& \text{ hence } \frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$$

$\therefore v$ is harmonic which is imaginary part of $f(z)$ $\therefore$ Real & imaginary parts of analytic funⁿ of $f(z)$ are harmonic

ii) Polar form: Let $f(z)=u(r,\theta)+iv(r,\theta)$ be analytic we shall show that,

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0 \quad \&$$

$$\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial \theta^2}=0$$

ST, $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0$ &

$$\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial \theta^2}=0$$

Since $f(z)$ is analytic,

$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}\rightarrow(1) \quad \& \quad \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}\rightarrow(2)$$

diff (1) wrt r & (2) wrt θ,

$$\frac{\partial^2 u}{\partial r^2}=-\frac{1}{r^2}\frac{\partial v}{\partial \theta}+\frac{1}{r}\frac{\partial^2 v}{\partial\theta\partial r}\rightarrow(3) \quad \&$$

$$\frac{\partial^2 v}{\partial\theta\partial r}=-\frac{1}{r}\frac{\partial^2 u}{\partial \theta^2}$$

$\therefore$ (3) can be written as, $\frac{\partial^2 u}{\partial r^2}=-\frac{1}{r^2}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r}\left(-\frac{1}{r}\frac{\partial^2 u}{\partial \theta^2}\right)$

$$\frac{\partial^2 u}{\partial r^2}=-\frac{1}{r}\frac{\partial u}{\partial r}-\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

$$\Rightarrow \frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$$

$\therefore u$ is harmonic which is real part of $f(z)$

Now, diff (1) wrt θ & (2) wrt r,

$$\frac{\partial^2 v}{\partial r\partial \theta}=\frac{1}{r}\frac{\partial^2 v}{\partial \theta^2} \quad \&$$

$$\frac{\partial^2 v}{\partial r^2}=\frac{1}{r^2}\frac{\partial u}{\partial \theta}-\frac{1}{r}\frac{\partial^2 u}{\partial r\partial \theta}\rightarrow(4)$$

$$\frac{\partial^2 v}{\partial r^2} = \frac{1}{r^2}\left(-r\frac{\partial v}{\partial \theta}\right) - \frac{1}{r}\left(\frac{1}{r}\frac{\partial^2 v}{\partial \theta^2}\right)$$

$$\frac{\partial^2 v}{\partial r^2} = -\frac{1}{r}\frac{\partial v}{\partial \theta} - \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}$$

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0$$

∴ v is harmonic. Thus real & imaginary parts of an analytic funⁿ are harmonic.

7) If $f(z) = u+iv$ is analytic then families of curves $u(x,y)=c_1$ & $v(x,y)=c_2$, c_1 & c_2 being constants. PT these curves intersect each other orthogonally.

Sol: If $f(z)=u(x,y)+iv(x,y)$ is analytic then the family of curves $u(x,y)=c_1$ & $v(x,y)=c_2$ where c_1 & c_2 being constants, intersect with each other orthogonally.

Proof: Consider the family of the curve $u(x,y)=c_1$ & $v(x,y)=c_2$ where c_1 & c_2 are constants. Then y is a function of x on each of the curves differentiating $u(x,y)=c_1$ wrt x, $\frac{du}{dx}=0$

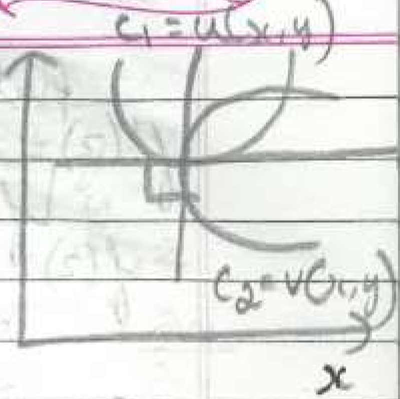
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx} = 0$$

$$\frac{\partial u}{\partial y}\frac{dy}{dx} = -\frac{\partial u}{\partial x}$$

$$\frac{dy}{dx} = \frac{-\partial u/\partial x}{\partial u/\partial y} = m$$

diff $v(x,y)=c_2$ wrt x,

$$\frac{dv}{dx} = 0$$



$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{dy}{dx} = 0$$

$$\frac{\partial v}{\partial y}\frac{dy}{dx} = -\frac{\partial v}{\partial x}$$

$$\frac{dy}{dx} = \frac{-\partial v/\partial x}{\partial v/\partial y} = m_2$$

Now, $m_1 \times m_2 = \frac{-\partial u/\partial x}{\partial u/\partial y} \times \frac{-\partial v/\partial x}{\partial v/\partial y} \rightarrow (1)$

But $f=u(x,y)+iv(x,y)$ is analytic we have CR eqⁿs, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

using these in (1),

$$m_1 \times m_2 = \frac{-\partial v/\partial y}{\partial u/\partial y} \cdot \frac{\partial u/\partial y}{\partial v/\partial y} = -1$$

hence the curves $u(x,y)=c_1$ & $v(x,y)=c_2$ intersect orthogonally at every point of intersection.

8) Find the analytic function $f(z)$ whose imaginary part is $e^x(x\sin y + y\cos y)$

Sol: $v = e^x x\sin y + e^x y\cos y$

diff wrt x & y,

$$\frac{\partial v}{\partial x} = e^x(1\times \sin y + 0) + e^x(x\sin y + y\cos y)$$

$$= e^x(\sin y + x\sin y + y\cos y)$$

$$\frac{\partial v}{\partial y} = e^x(x\cos y + y(-\sin y) + 1\times\cos y)$$

$$\frac{\partial v}{\partial y} = e^x(x\cos y - y\sin y + \cos y)$$

$$\boxed{f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}}$$

$f'(z) = e^x[x\cos y - y\sin y + \cos y] + i[e^x(\sin y +$

Put $x = z$ & $y = 0$

$f'(z) = e^z\{z-0+1\} + ie^z[0+0+0]$

$\boxed{f'(z) = e^z . z}$

$f(z) = \int z e^z dz$

$= ze^z - 1 \times e^z + 0 . e^z$

$= ze^z - e^z = (z-1)e^z$

$\boxed{f(z) = e^z(z-1)}$

to find u, $z = x+iy$

$u+iv = e^{(x+iy)}((x+iy)-1)$

$= e^x e^{iy}(x+iy-1)$

$= e^x[\cos y + i\sin y](x+iy-1)$

$= [e^x\cos y + ie^x\sin y](x+iy-1)$

$\cancel{= xe^x\cos y = ixe^x\sin y}$

$\cancel{+ iye^x\cos y + e^x y\sin y - e^x\cos y}$

$\cancel{+ ie^x\sin y}$

$\cancel{= xe^x\cos y - e^x\cos y + iy e^x\cos y + e^x y\sin y}$

$\cancel{= e^x(x-1)\cos y + i(x-1)e^x\sin y + i}$

$u+iv = [e^x\cos y + ie^x\sin y][(x-1)+iy]$

$= e^x(x-1)\cos y + ie^x(x-1)\sin y$

$+ iye^x\cos y - e^x y\sin y$

equating real & imaginary parts,

$u = e^x(x-1)\cos y - e^x y\sin y$

$v = e^x(x-1)\sin y + ye^x\cos y$

$u = e^x[(x-1)\cos y - y\sin y]$

$u = e^x[x\cos y - \cos y - y\sin y]$

is the required ~~imaginary~~ real part of $f(z)$

9) Show that $u = \log\sqrt{x^2+y^2}$ is harmonic & find analytic function $f(z)$

Sol: $u = \log\sqrt{x^2+y^2} = \log(x^2+y^2)^{1/2} = \frac{1}{2}\log(x^2+y^2)$

$\frac{\partial u}{\partial x} = \frac{1}{\cancel{2}} \cdot \frac{1}{x^2+y^2} \cancel{2}x \Rightarrow \frac{\partial u}{\partial x} = \frac{x}{x^2+y^2}$

$\frac{\partial u}{\partial y} = \frac{1}{\cancel{2}} \frac{1}{x^2+y^2} \cancel{2}y \Rightarrow \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2}$

$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ $\qquad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ (CR eqn)

$f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$

$= \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

put $x = z$ & $y = 0$ we have,

$f'(z) = \frac{z}{z^2+0} - i(0) = \frac{1}{z}$

$f(z) = \int \frac{1}{z} dz + c$

$\boxed{f(z) = \log z + c}$

10) Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and find its harmonic conjugate, also find the corresponding analytic funⁿ $f(z)$

Sol: $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x$; $\frac{\partial^2 u}{\partial x^2} = 6x + 6$

$\frac{\partial u}{\partial y} = -6xy - 6y$; $\frac{\partial^2 u}{\partial y^2} = -6x - 6$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 6 - 6x - 6 = 0$ Thus u is harmonic

Now consider $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

$\frac{\partial v}{\partial y} = 3x^2 - 3y^2 + 6x$; $\frac{\partial v}{\partial x} = -(-6xy - 6y) = 6xy + 6y$

$V = \int (3x^2 - 3y^2 + 6x)\,dy + f(x)$

$V = 3x^2 y - y^3 + 6xy + f(x)$

also, $V = \int (6xy + 6y)\,dx + g(y) \rightarrow ①$

$V = 3x^2 y + 6xy + g(y) \rightarrow ②$

from ① & ②, $f(x) = 0$ & $g(y) = -y^3$

$\therefore V = 3x^3 y - y^3 + 6xy$ is the harmonic conjugate

The analytic $f(z) = u + iv$

$f(z) = (x^3 - 3xy^{20} + 3x^2 - 3y^2 + 1) + i(3x^2 y - y^3 + 6xy)$

put $x = z,\ y = 0$

$\boxed{f(z) = z^3 + 3z^2 + 1}$

11) Determine the analytic function $f(z) = u + iv$ if $u + v = \frac{1}{r^2}(\cos 2\theta - \sin 2\theta)$

$\therefore$ $\boxed{f'(z) = e^{-i\theta}\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right)}$

$\frac{\partial u}{\partial r} + \frac{\partial v}{\partial r} = \frac{-2}{r^3}(\cos 2\theta - \sin 2\theta)$

diff w.r.t θ,

$\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial \theta} = \frac{1}{r^2}[-2\sin 2\theta - 2\cos 2\theta]$

CR eqn's are,

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \Rightarrow \frac{\partial v}{\partial \theta} = r\frac{\partial u}{\partial r}$

$\frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta} \Rightarrow \frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$

$-r\frac{\partial v}{\partial r} + r\frac{\partial u}{\partial r} = \frac{-2}{r^2}(\sin 2\theta + \cos 2\theta)$

$-\frac{\partial v}{\partial r} + \frac{\partial u}{\partial r} = \frac{-2}{r^3}(\cos 2\theta + \sin 2\theta)$

$\frac{\partial u}{\partial r} - \frac{\partial v}{\partial r} = \frac{-2}{r^3}(\cos 2\theta + \sin 2\theta) \rightarrow ②$

① & ② are simultaneous eqn

$\therefore ① + ②$, $2\frac{\partial u}{\partial r} = \frac{-2}{r^3}[\cos 2\theta - \cancel{\sin 2\theta} + \cos 2\theta + \cancel{\sin 2\theta}]$

$\cancel{2}\frac{\partial u}{\partial r} = \frac{-\cancel{2}}{r^3} \times 2\cos 2\theta$

$\boxed{\frac{\partial u}{\partial r} = \frac{-2}{r^3}\cos 2\theta}$

$① - ②, \Rightarrow \cancel{2}\frac{\partial v}{\partial r} = \frac{-\cancel{2}}{r^3}[-2\sin 2\theta]$

$\boxed{\frac{\partial v}{\partial r} = \frac{2\sin 2\theta}{r^3}}$

$f'(z) = e^{-i\theta}\left[\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right]$

$= e^{-i\theta}\left[\frac{-2}{r^3}\cos 2\theta + i\frac{2\sin 2\theta}{r^3}\right]$

put $r = z,\ \theta = 0$

$= e^{-i(0)}\left[\frac{-2}{r^3}\cos 2\theta + i\frac{2}{r^3}\sin 2\theta\right]$

put $r = z,\ \theta = 0$

$= e^{-i(0)}\left[\frac{-2}{z^3}\cos 2(0) + i\frac{2}{z^3}\sin 2(0)\right]$

$= \frac{-2}{z^3} \Rightarrow \boxed{f'(z) = \frac{-2}{z^3}}$

$f'(z) = -2z^{-3}$

$f(z) = -2\int z^{-3}\,dz$

$= -2\left[\frac{z^{-2}}{-2}\right] = z^{-2}$

2) If $f(z)$ is analytic, show that
$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2$$

: Let $f(z)=u+iv$ be analytic

$|f(z)| = \sqrt{u^2+v^2}$

$|f(z)|^2 = u^2+v^2 = \phi$

To prove that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\phi = 4|f'(z)|^2$

That is to prove that,
$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2} = 4|f'(z)|^2$$

Consider $\phi = u^2+v^2$ & differentiate wrt x partially.
$$\frac{\partial\phi}{\partial x} = 2u\frac{\partial u}{\partial x}+2v\frac{\partial v}{\partial x}$$
$$= 2[uu_x+vv_x]$$

differentiating wrt x again we get,
$$\frac{\partial^2\phi}{\partial x^2} = 2[uu_{xx}+u_x^2+vv_{xx}+v_x^2]$$

III^{ly} we can also get,
$$\frac{\partial^2\phi}{\partial y^2} = 2[uu_{yy}+u_y^2+vv_{yy}+v_y^2]$$

adding ① & ② we have
$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2} = 2[u(u_{xx}+u_{yy})+v(v_{xx}+v_{yy})+u_x^2+v_x^2+u_y^2+v_y^2+\ldots]$$

Since $f(z)=u+iv$ is analytic, u & v are harmonic.

Hence $u_{xx}+u_{yy}=0$

$v_{xx}+v_{yy}=0$. Further we also have

CR eq^{n}s $v_y=u_x$ & $u_y=-v_x$

$\therefore \phi_{xx}+\phi_{yy} = 2[u(0)+v(0)+u_x^2+v_x^2+(-v_x)^2$

$$\phi_{xx}+\phi_{yy} = 2[2u_x^2+2v_x^2] = 4(u_x^2+v_x^2)$$

$f'(z) = u_x+iv_x$

$|f'(z)| = \sqrt{u_x^2+v_x^2}$ or $|f'(z)|^2 = u_x^2+v_x^2$

$\therefore$ Using this, we get
$$\phi_{xx}+\phi_{yy} = 4|f'(z)|^2$$
Hence proved.

13) For an ~~inco~~ incompressible fluid there exists an analytic fun^n $F(z)=\phi(x,y)+i\varphi(x,y)$. Describe the terms & what if $\phi(x,y)$ & $\varphi(x,y)$ are constants. It is the function is still analytic.

Sol: In any 2D incompressible fluid, the velocity happens to be the gradient of a scalar harmonic fun^n. The scalar fun^n is called as velocity potential & is usually denoted by ϕ. The harmonic conjugate of this fun^n is called the stream fun^n for the flow & is usually denoted by ψ. The analytic fun^n for which ϕ is the real part & ψ is the imaginary part is called the complex potential for the flow & is usually denoted by $w=\phi+i\psi$

consequently $\phi=c_1$ & $\psi=c_2$ are orthogonal. The curves ψ is constant is called stream lines & $\phi=c_1$ are called equipotential lines

14) In a 2D fluid flow if velocity potential $\phi = 3x^2y-3y^3$. Find the stream

Sol: $\phi = 3x^2y - y^3$

$\frac{\partial\phi}{\partial x} = 6xy \quad \frac{\partial\phi}{\partial y} = 3x^2 - 3y^2$

CR eqn; $\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$

$\frac{\partial\psi}{\partial x} = -\frac{\partial\phi}{\partial y}$

$\frac{\partial\psi}{\partial y} = 6xy = \frac{\partial\phi}{\partial x}$

integrating on b.s w.r.t y,

$\psi = 6\int xy\,dy = \frac{\cancel{6}^{3}\,xy^2}{\cancel{2}} + f(x)$

$\boxed{\psi = 3xy^2 + f(x)} \longrightarrow ①$

$\frac{\partial\phi}{\partial y} = \frac{-\partial\psi}{\partial x} = 3x^2 - 3y^2$

$-\frac{\partial\psi}{\partial x} = 3y^2 - 3x^2$

integrating w.r.t x,

$\psi = 3y^2x - \frac{\cancel{3}x^3}{\cancel{3}} + g(y)$

$\boxed{\psi = 3y^2x - x^3 + g(y)} \longrightarrow ②$

$\therefore$ Comparing ① & ②,

$f(x) = -x^3$ & $g(y) = 0$

$\boxed{\therefore \psi = 3y^2x - x^3}$ is the stream funn

Complex function $f(z) = \phi + i\psi$

$f(z) = (3x^2y - y^3) + i(3xy^2 - x^3)$

put $x = z$ & $y = 0$,

$f(z) = (0-0) + i(0 - z^3)$

15) If $F(z) = \phi(x,y) + i\psi(x,y)$ represent the complex potential of an electrostatics field, where $\psi = (x^2 - y^2) + \left[\frac{x}{x^2+y^2}\right]$; find $f(z)$ & hence determine ϕ.

Sol: $\frac{\partial\psi}{\partial x} = 2x + \frac{(x^2+y^2)(1) - x\cdot 2x}{(x^2+y^2)^2}$

$= 2x + \frac{y^2 - x^2}{(x^2+y^2)^2}$

$\frac{\partial\psi}{\partial y} = -2y + \frac{(x^2+y^2)(0) - x \times 2y}{(x^2+y^2)^2}$

$= -2y - \frac{2xy}{(x^2+y^2)^2}$

Consider $f'(z) = \phi_x + i\psi_x$ but $\phi_x = \psi_y$.

$f'(z) = \psi_y + i\psi_x$

put $x = z$, $y = 0$ we have,

$f'(z) = [\psi_y]_{(z,0)} + i[\psi_x]_{(z,0)}$

$f'(z) = 0 + i\left(2z + \frac{-z^2}{(z^2)^2}\right)$

$f'(z) = i\left(2z - \frac{1}{z^2}\right)$ integrating on b.s w.r.t z

$f(z) = i\int\left(2z - \frac{1}{z^2}\right)dz + c$

$= i\left(z^2 + \frac{1}{z}\right) + c$

$\boxed{f(z) = i\left(z^2 + \frac{1}{z}\right) + c}$

$\phi = ?$, seperating real & imaginary parts,

$\phi + i\psi = i\left[(x+iy)^2 + \frac{1}{x+iy}\right] + c$

$= i\left[(x^2+i^2y^2+2xiy) + \frac{(x-iy)}{(x+iy)(x-iy)}\right]+c$

$= i\left[(x^2-y^2)+2xiy\right]+i\left[\frac{x-iy}{x^2+y^2}\right]+c$

$= i(x^2-y^2) - 2xy + \frac{ix}{x^2+y^2} + \frac{y}{x^2+y^2} + c$

$= \left(-2xy + \frac{y}{x^2+y^2}\right) + i\left(x^2-y^2+\frac{x}{x^2+y^2}\right)+c$

equating real & imaginary parts,

$$\boxed{\begin{aligned}\phi &= -2xy + \frac{y}{x^2+y^2}\\ \psi &= x^2-y^2+\frac{x}{x^2+y^2}\end{aligned}}$$

1AM19IS058 - Meghana PS

1 ✓
2 ✓
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18/04/20

ASSIGNMENT - II

Module 2 : Conformal transformations
Module 3 : Probability distribution

1) Discuss the transformation $w = z + \frac{1}{z}$

Sol: Given, $w = f(z) = z + \frac{1}{z}$ is analytic, $z \neq 0$

$\therefore f(z)$ is differentiable

$f'(z) = z + \frac{1}{z}$, $z \neq 0$

$= 1 - \frac{1}{z^2}$, $z \neq 1$

let $z = re^{i\theta}$

$\therefore$ wkt, $w = f(z) = u + iv = z + \frac{1}{z}$

$u + iv = re^{i\theta} + \frac{1}{re^{i\theta}}$

$u + iv = re^{i\theta} + \frac{1}{r}e^{-i\theta}$

$u + iv = r(\cos\theta + i\sin\theta) + \frac{1}{r}(\cos\theta - i\sin\theta)$

$= r(\cos\theta + ir\sin\theta) + \left(\frac{1}{r}\cos\theta - i\frac{1}{r}\sin\theta\right)$

$u + iv = \left(r+\frac{1}{r}\right)\cos\theta + i\left(r-\frac{1}{r}\right)\sin\theta$

$u = \left(r+\frac{1}{r}\right)\cos\theta \rightarrow (1)$

$v = \left(r-\frac{1}{r}\right)\sin\theta \rightarrow (2)$

$\therefore (1) \Rightarrow \frac{u}{\left(r+\frac{1}{r}\right)} = \cos\theta \rightarrow (3)$

$(2) \Rightarrow \frac{v}{\ldots} = \sin\theta \rightarrow (4)$

Sol: Given, $z_1 = 1, z_2 = i, z_3 = -1$ & images are $w_1 = i, w_2 = 0, w_3 = -i$

wkt from the bilinear transformation,

$$\boxed{\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}}$$

but Let $\boxed{w = \frac{az+b}{cz+d}}$ be the required bilinear transformation.

$\rightarrow z=1, w=i \Rightarrow i = \frac{a+b}{c+d}$

$a+b = ic+id \Rightarrow a+b-ic-id=0 \rightarrow (1)$

$\rightarrow z=i, w=0 \Rightarrow 0 = \frac{ai+b}{ci+d}$

$ai+b=0 \rightarrow (2)$

$\rightarrow z=-1, w=-i$, gives $-i = \frac{-a+b}{-c+d}$

$-a+b = +ic-id$

$-a+b-ic+id=0 \rightarrow (3)$

adding eq^n (1) & (3) $\Rightarrow (1)+(3)$,

$2b-2ci=0 \Rightarrow b-ci=0 \rightarrow (4)$

Let (2) & (4) be written as,

$ia+1b+0c=0$

$0a+1b-ic=0$

applying cross multiplication we have,

$$\frac{a}{\begin{vmatrix}1 & 0\\ 1 & -i\end{vmatrix}} = \frac{-b}{\begin{vmatrix}i & 0\\ 0 & -i\end{vmatrix}} = \frac{c}{\begin{vmatrix}i & 1\\ 0 & 1\end{vmatrix}}$$

i.e., $\frac{a}{-i} = \frac{-b}{-i^2} = \frac{c}{i}$ or

$\frac{a}{-i} = \frac{b}{-1} = \frac{c}{i} = K$ (where K is a constant)

$\therefore \boxed{a=-iK, b=-K, c=iK}$

Substituting all these in (1),

$-iK-K+K-di=0$ or

$-(di+iK)=0 \Rightarrow \boxed{d=-K}$

now substituting the values of a, b, c, d in bilinear transformation eq^n,

$$w = \frac{-iKz-K}{iKz-K} = \frac{-K(1+iz)}{-K(1-iz)}$$

$\therefore \boxed{w = \frac{1+iz}{1-iz}}$ is the required bilinear transformation.

To find the image of $|z|<1$,

consider $z=-1$

$$\therefore w = \frac{1-i}{1+i} = \frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{(1-i)^2}{1-i^2}$$

$$= \frac{1+i^2-2i}{2}$$

$\boxed{w=-i}$

$\therefore$ the image of $|z|<1$ is $w=-i$.

3) Discuss the transformation $w=e^z$

Sol: Given, $w=f(z)=e^z$ is analytic

$\therefore f(z)$ is differentiable

$\therefore f'(z)=e^z, \quad z \neq 0$

we have $w=f(z)=u+iv=e^z \rightarrow (1)$

let $z=x+iy$

$\therefore (1) \Rightarrow u+iv = \cancel{e^x+iy}\ e^{x+iy}$

$u+iv = e^x \cdot e^{iy}$

$u+iv = e^x(\cos y + i\sin y)$

$u+iv = e^x\cos y + ie^x\sin y$

$\therefore u = e^x\cos y \rightarrow (2)$ & $v = e^x\sin y \rightarrow (3)$

By eliminating 'y' we get,

$②^2 + ③^2 \Rightarrow u^2+v^2 = (e^x\cos y)^2 + (e^x \sin y)^2$

$u^2+v^2 = e^{2x}(\cos^2 y + \sin^2 y)$

$\therefore u^2+v^2 = e^{2x} \longrightarrow ④$

By eliminating x from the eq^n ② & ③,

$\frac{③}{②} \Rightarrow \frac{v}{u} = \frac{e^x \sin y}{e^x \cos y} = \frac{\sin y}{\cos y} = \tan y$

$\frac{v}{u} = \tan y$

$\Rightarrow v = u \cdot \tan y \longrightarrow ⑤$

Case 1: Let $x = c_1$ = constant, we have by eq^n ④

$u^2+v^2 = e^{2x} \Rightarrow u^2+v^2 = e^{2c_1} = r_1$

$\Rightarrow u^2+v^2 = r^2 \longrightarrow ⑥$

r represents a circle having centre as origin with radius "r".

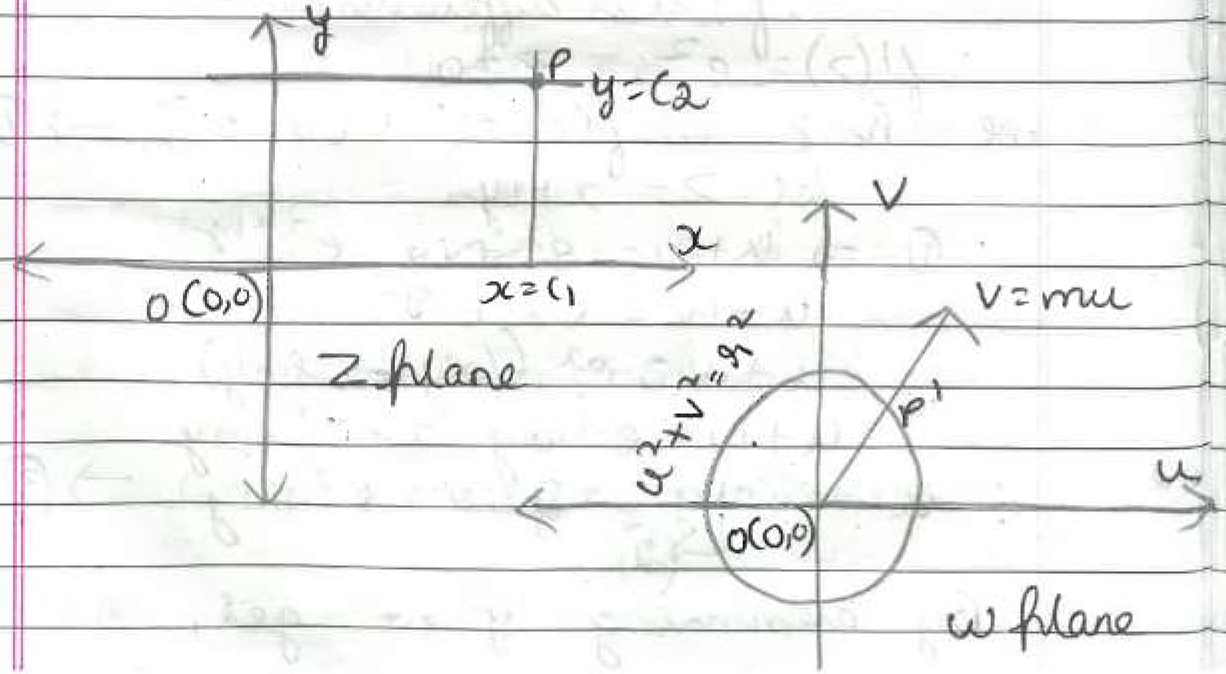
Case 2: Let $y = c_2$ = constant, we have

eq^n ⑤, $v = \tan y \cdot u$ $\quad \because y = c_2$

$v = \tan c_2 \, u$

$v = mu \longrightarrow ⑦$ where $m = \tan c_2$ = slope

represents a straight line passing through the origin having the slope 'm'.



4) Find the bilinear transformation that transforms $z = -1, i, 1$ on to the points $w = 1, i, -1$. Also find invariant points

Sol: Let $z_1 = -1$, $z_2 = i$, $z_3 = 1$ & images on them are $w_1 = 1$, $w_2 = i$, $w_3 = -1$

The required bilinear transformation is,

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

$$\frac{(w-1)(i+1)}{(w+1)(i-1)} = \frac{(z+1)(i-1)}{(z-1)(i+1)}$$

$$\frac{(w-1)}{(w+1)} = \frac{(i-1)}{(i+1)} \times \frac{(i-1)}{(i+1)} \frac{(z+1)}{(z-1)}$$

$$\frac{(w-1)}{(w+1)} = \frac{(i-1)^2}{(i+1)^2} \times \frac{z+1}{z-1}$$

$$= \frac{(i^2+1-2i)}{(i^2+1+2i)} \times \frac{z+1}{z-1}$$

$$= \left(\frac{-2i}{2i}\right) \times \frac{z+1}{z-1}$$

$$\frac{(w-1)}{(w+1)} = -\frac{(z+1)}{(z-1)}$$

$$\frac{(w-1)}{(w+1)} = \frac{(1+z)}{(1-z)}$$

$(w-1)(1-z) = (w+1)(1+z)$

$w - wz - 1 + z = w + wz + 1 + z$

$-2wz - 2 = 0$

$-\cancel{2} wz = \cancel{2}$

$\boxed{w = \frac{-1}{z}}$ is the required bilinear transformation

To find the invariant points,

$w = \frac{-1}{z}$

but $w = z$

$$Z = \frac{-1}{Z}$$
$$Z^2 = -1$$
$$Z = \sqrt{-1}$$
but $\sqrt{-1} = i$

hence $\boxed{Z = i}$

now $\omega = \frac{-1}{Z}$ but $Z = i$,

$$\omega = \frac{-1}{i} = -(-i) = +i$$

$\therefore \boxed{\omega = i}$

$\therefore$ at $Z = i$, the image $\omega = i$

hence they are the invariant points.

5) Suppose a random variable takes the values $-3, -1, 2$ & 5 with respective probabilities, $\frac{2K-3}{10}, \frac{K-2}{10}, \frac{K-1}{10}, \frac{K+1}{10}$.

Find the value of K &

i) $P(-3 \le x \le 2)$ & ii) $P(x \le 2)$

Sol:

x	-3	-1	2	5
$P(x)$	$\frac{2K-3}{10}$	$\frac{K-2}{10}$	$\frac{K-1}{10}$	$\frac{K+1}{10}$

$\boxed{\sum_{i=1}^{n} P(x_i) = 1}$ where $n = 4$

$$\sum_{i=1}^{4} P(x_i) = P(x_1) + P(x_2) + P(x_3) + P(x_4) = 1$$
$$= \frac{2K-3}{10} + \frac{K-2}{10} + \frac{K-1}{10} + \frac{K+1}{10} = 1$$
$$= \frac{2K-3+K-2+K-1+K+1}{10} = 1$$
$$= \frac{5K-5}{10} = 1$$

$K - 1 = 2 \Rightarrow K = 2+1 \Rightarrow \boxed{K = 3}$

$K = 3 \;\therefore \sum_{i=1} P(x_i) = 1$ hence property is satisfied

i) $P(-3 \le x \le 2) \Rightarrow P(x=-3) + P(x=-1) + P(x=2)$
$$= \frac{2K-3}{10} + \frac{K-2}{10} + \frac{K-1}{10}$$
$$= \frac{2(3)-3}{10} + \frac{3-2}{10} + \frac{3-1}{10}$$
$$= \frac{3}{10} + \frac{1}{10} + \frac{2}{10}$$

$\boxed{\therefore P(-3 \le x \le 2) = \frac{3}{5} = 0.6}$

ii) $P(x \le 2) = 1 - P(x > 2)$ ✗
$$= 1 - P(X = 5)$$
$$= 1 - \left(\frac{K+1}{10}\right) = 1 - \left(\frac{3+1}{10}\right)$$

$\boxed{P(x \ge 2) = \frac{3}{5} = 0.6}$

or

$P(x \le 2) = P(x=-3) + P(x=-1) + P(x=2)$ as done above,

$\boxed{P(x \le 2) = 0.6}$

$\therefore$ the value of $K = 3$,

$P(-3 \le x \le 2) = 0.6$ & $P(x \le 2) = 0.6$

Sol: $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\cancel{\int_{-\infty}^{0} f(x)dx} + \int_0^1 f(x)dx + \cancel{\int_1^{\infty} f(x)dx} \to 0$

$= \int_0^1 f(x)\,dx = \int_0^1 Ke^{-x}\,dx$

$= K\int_0^1 e^{-x}dx = K-[e^{-x}]_0^1 = -K(e^{-1}-e^{-0})$

$= -K(e^{-1}-1) = 1 \Rightarrow -K\left[\frac{1}{e}-1\right] = 1$

~~$-Ke^{-1}+K=1$~~

$-K\left[\frac{1-e}{e}\right] = 1$

$K\left[\frac{e-1}{e}\right] = 1 \Rightarrow \boxed{K = \frac{e}{e-1}}$

$\mu = \int_{-\infty}^{\infty} x\left(\frac{e}{e-1}\right)e^{-x}dx$

~~$\frac{e}{e-1}\int_{-\infty}^{\infty} x\,dx = \frac{e}{e-1}\left[\int_{-\infty}^{0} x\,dx + \int_0^1 x\,dx + \int_1^{\infty} f(x)dx\right]$~~

~~$\mu = \frac{e}{e-1}\left[\frac{x^2}{2}\right]_0^1 = \frac{e}{e-1}\left[\frac{1^2}{2}-0\right]$~~

~~$\therefore \boxed{\mu = \frac{e}{2(e-1)}}$~~

~~$\boxed{\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2}$~~

~~$= \int_{-\infty}^{\infty} x^2$~~

$\mu = \int_{-\infty}^{\infty} x\left(\frac{e}{e-1}\right)e^{-x}dx$

$= \left(\frac{e}{e-1}\right)\left[\cancel{\int_{-\infty}^{0} xe^{-x}dx} + \int_0^1 xe^{-x}dx + \cancel{\int_1^{\infty} xe^{-x}dx}\right]$

$= \left(\frac{e}{e-1}\right)\left[\frac{-x^2}{2}e^{-x}\right]_0^1 = -\left(\frac{e}{e-1}\right)\left(\frac{1^2}{2}e^{-1} - \frac{0^2 e^{-0}}{2}\right)$

$= -\left(\frac{e}{e-1}\right)\left(\frac{1}{2e}\right) = \frac{-1}{e-1} = \frac{1}{1-e}$

$\boxed{\mu = \frac{1}{1-e}}$

$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$= \int_{-\infty}^{\infty} x^2\left(\frac{e}{e-1}\right)e^{-x}dx - \left(\frac{1}{1-e}\right)^2$

$= \left(\frac{e}{e-1}\right)\int_0^1 x^2 e^{-x}dx - \left[\frac{1}{1-e}\right]^2$

$= \left(\frac{e}{e-1}\right)\left[\frac{-x^3}{3}e^{-x}\right]_0^1 - \left[\frac{1}{1-e}\right]^2$

$= -\left(\frac{e}{e-1}\right)\left(\frac{1^3}{3}e^{-1} - \frac{0^3}{3}e^{-0}\right) - \frac{1}{(1-e)^2}$

$= -\left(\frac{e}{e-1}\right)\left(\frac{1}{3}e^{-1}\right) - \frac{1}{(1-e)^2}$

$= \frac{-1}{3(e-1)} - \frac{1}{(1-e)^2}$

$\boxed{\therefore \sigma^2 = -\left[\frac{1}{3(e-1)} + \frac{1}{(1-e)^2}\right]}$

$\sigma = $

6) Out of 800 families with 4 children each how many families would be expected to have i) 2 boys & 2 girls, ii) atleast one boy iii) no girl, iv) atmost 2 girls by assuming equal probability of boys & girls

Sol: p - probability of having girls $= 1/2$
q - probability of having boys $= 1/2$
Let x denote no. of boys in the family

$\boxed{P(X=r) = {}^nC_r p^r q^{n-r}}$ where $n=4$,

$= {}^4C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{4-r}$

$= {}^4C_r \left(\frac{1}{2}\right)^{\cancel{r}+4-\cancel{r}}$

$P(r) = {}^4C_r \left(\frac{1}{2}\right)^4$

i) 2 boys & 2 girls [as no of boys is equal to number of girls, equate it only once]

$P(X=2) = {}^4C_2 \left(\frac{1}{2}\right)^4 = 0.375$

$\therefore$ out of 800 families,
$800 \times 0.375 = \underline{300}$ families have 2 boys & 2 girls.

ii) atleast one boy,
$P(x \geq 1) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$ or
$P(x \geq 1) = 1 - P(x < 1)$
$= 1 - P(x=0)$
but $P(x=0) = {}^4C_0 \left(\frac{1}{2}\right)^4 = \frac{1}{16} = \underline{0.0625}$

~~out of 800 families, 0.0625 X 800~~
~~= 50~~

$P(x \geq 1) = 1 - 0.0625 = 0.9375$
$\therefore$ out of 800 families, 800×0.9375
$= 750$
$\therefore$ 750 family have atleast 1 boy.

iii) no girl $\Rightarrow P(x=0) = {}^4C_0 \left(\frac{1}{2}\right)^4 = 0.0625$

$\therefore$ out of 800 families, $800 \times 0.0625 = 50$
$\therefore$ 50 families are have no girl

iv) atmost 2 girls,
$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$
$P(x=0) = {}^4C_0 \left(\frac{1}{2}\right)^4 = 0.0625$

$P(x=1) = {}^4C_1 \left(\frac{1}{2}\right)^4 = 0.25$

$P(x=2) = {}^4C_2 \left(\frac{1}{2}\right)^4 = 0.375$

$\therefore P(x \leq 2) = 0.0625 + 0.25 + 0.375$
$P(x \leq 2) = 0.6875$
$\therefore$ out of 800 families, 800×0.6875
$= 550$
$\therefore$ 550 familes have atmost 2 girls.

10) $f(x) = \begin{cases} \alpha e^{-\alpha x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$\alpha = \frac{1}{\mu} \Rightarrow \boxed{\mu = \frac{1}{\alpha}} \Rightarrow \alpha = 100$

average sale amounting is Rs.100 $= \mu = \frac{1}{100}$

$\therefore$ ~~$\mu = 0.01$~~ $\mu = \frac{1}{\alpha} = 0.01$

$\boxed{\mu = 0.01}$

10)

Sol: $\alpha = 100 = \frac{1}{\mu} \Rightarrow \mu = \frac{1}{100} = 0.01$

$\boxed{\mu = 100, \ \alpha = 0.01}$

profit = 0.08

$\boxed{f(x) = \int_{-\infty}^{\infty} \begin{cases} \alpha e^{-\alpha x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}}$

$P(x > 30) = 1 - P(x \leq 30)$

$P(x \leq 30) = \int_0^{30} 0.01 e^{-0.01x} \, dx$

$= \cancel{0.01} \left[\frac{e^{-0.01x}}{-\cancel{0.01}} \right]_0^{30}$

$= -[e^{-0.01(30)} - \cancel{e^{-0.01(0)}}]$

$P(x \leq 30) = -[0.7408]$

$\therefore \ P(x > 30) = 1 - (-0.7408)$

$= 1 + 0.7408$

$= 1.7408$

$\therefore$ The probability that net profit exceeds Rs 30 on a day is $\underline{1.7408}$

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Date
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ASSIGNMENT: 03 - MODULE 02 - CONFORMAL TRANSFORMATIONS.
MODULE 05 - JOINT PROBABILITY DISTRIBUTIONS & SAMPLING THEORY

1) The joint probability distribution by two random variable X & Y are given below

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find i) the marginal distribution of X & Y
ii) Covariance of X & Y
iii) $P(X, Y)$, iv) verify X & Y are independent or not

Sol: Marginal distribution of X & Y are got by adding all the respective row entries & the respective column entries.

x_i	1	2
$f(x_i)$	0.6	0.4

y_j	-2	-1	4	5
$g(y_j)$	0.3	0.3	0.1	0.3

$\boxed{\mu_x = E(X) = \sum_i x_i f(x_i)}$

$= (1)(0.6) + (2)(0.4) = 1.4$

$\boxed{\mu_y = E(Y) = \sum_j y_j \cdot g(y_j)}$

$= (-2)(0.3) + (-1)(0.3) + (4)(0.1) + (5)(0.3)$
$= -0.6 - 0.3 + 0.4 + 1.5$
$= 1$

$\boxed{\mu_y = 1}$

$E(XY) = \sum_{i,j} x_i y_j J_{ij}$
$= (1)(-2)(0.1) + (1)(-1)(0.2) + (1)(4)(0) +$
$(1)(5)(0.3) + (2)(-2)(0.2) + (2)(-1)(0.1)$
$+ (2)(4)(0.1) + (2)(5)(0)$
$= -0.2 - 0.2 + 0 + 1.5 - 0.8 - 0.2 + 0.8 + 0$
$= 0.9.$

$\therefore \boxed{E(X) = 1.4,\ E(Y) = 1\ \&\ E(XY) = 0.9}$

$\boxed{\sigma_x^2 = E(X^2) - \mu_x^2}$

$E(X^2) = \sum_i x_i^2 f(x_i)$
$= (1)(0.6) + (4)(0.4)$
$= 2.2$
$= 2.2 - (1.4)^2$
$\boxed{\sigma_x^2 = 0.24}$ & $\boxed{\sigma_x = 0.49}$

$\boxed{\sigma_y^2 = E(Y^2) - \mu_y^2}$

$\boxed{E(Y^2) = \sum y_i^2 g(y_i)}$

$= (4)(0.3) + (1)(0.3) + (16)(0.1) + (25)(0.3)$
$= 10.6$
$\sigma_y^2 = 10.6 - (1)^2 = 9.6$
$\boxed{\sigma_y^2 = 9.6}$ & $\boxed{\sigma_y = 3.1}$

$cov(X,Y) = E(XY) - E(X)E(Y)$
$= 0.9 - (1.4)(1)$
$= -0.5$

$\boxed{cov(X,Y) = -0.5}$

Correlation of X & Y = $\boxed{\rho(X,Y) = \frac{cov(X,Y)}{\sigma_x \sigma_y}}$

$\rho(X,Y) = \frac{-0.5}{(0.49)(3.1)} = -0.3$

$\boxed{\rho(X,Y) = -0.3}$

If X & Y are independent random variables we must have $f(x_i)g(y_j) = J_{ij}$
It can be seen that $f(x_1)g(y_1)$
$= (0.6)(0.3)$
$= 0.18$ &

$J_{11} = 0.1$

$f(x_1)g(y_1) \neq J_{11}$
Similarly for others also the condition is not satisfied.
Hence we conclude that X & Y are dependent random variables.

2) A fair coin is tossed three times. Let X denote 0 or 1 according as a head or tail occurs on the first toss & let Y denote the number of heads which occur. Find i) the distribution of X & Y; ii) joint distribution of X & Y; iii) $cov(X,Y)$

The sample space S & the association of random variables X & Y is given by the following table

S	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
X	0	0	0	0	~~0~~ 1	1	1	1
Y	3	2	2	1	2	1	1	0

The probability distribution of X & Y is found as follows

$X = \{x_i\} = \{0, 1\}$ &

$Y = \{y_j\} = \{0, 1, 2, 3\}$

$P(X=0)$ is $4/8 = 1/2$

$P(X=1)$ is $4/8 = 1/2$

$P(Y=0)$ is $1/8$; $P(Y=2)$ is $3/8$

$P(Y=1)$ is $3/8$; $P(Y=3)$ is $1/8$

Thus we have the following probability distribution of X & Y

x_i	0	1
$f(x_i)$	1/2	1/2

y_j	0	1	2	3
$g(y_j)$	1/8	3/8	3/8	1/8

The joint distribution of X & Y is found by computing

$J_{ij} = P(X = x_i, Y = y_j)$ where we have,

$x_1 = 0 ; x_2 = 1$ & $y_1 = 0 ; y_2 = 1 ; y_3 = 2$, $y_4 = 3$

$J_{11} = P(X=0, Y=0) = 0$

$J_{12} = P(X=0, Y=1) = 1/8$ corresponding to the out come HTT

$J_{13} = P(X=0 ; Y=2) = 2/8 = 1/4$ out comes are HHT & HTH

$J_{14} = P(X=0, Y=3) = 1/$ [illegible]

$J_{21} = P(X=1, Y=0) = 1/8$ outcome is TTT

$J_{22} = P(X=1, Y=1) = 2/8 = 1/4$ out comes are THT, TTH

$J_{23} = P(X=1, Y=2) = 1/8$ outcome is THH

$J_{24} = P(X=1, Y=3) = 0$ since the outcome is impossible

The required joint probability distribution of X & Y is as follows

X \ Y	0	1	2	3	Sum
0	0	1/8	1/4	1/8	1/2
1	1/8	1/4	1/8	0	1/2
Sum	1/8	3/8	3/8	1/8	1

$$\boxed{\mu_X = E(X) = \sum_i x_i f(x_i)}$$

$$\mu_X = (0)(1/2) + (1)(1/2) = \boxed{1/2}$$

$$\boxed{\mu_Y = E(Y) = \sum_j y_j g(y_j)}$$

$$= (0)(1/8) + (1)(3/8) + (2)(3/8) + (3)(1/8)$$

$$= 12/8$$

$$\boxed{\mu_Y = \frac{3}{2}}$$

$$E(X,Y) = \sum_{i,j} x_i y_j J_{ij}$$

$$E(X,Y) = 0 + (0 + 1/4 + 2/8 + 0)$$

$$\boxed{E(X,Y) = 1/2}$$

$\sigma_X^2 = E(X^2) - \mu_X^2$ & $\sigma_Y^2 = E(Y^2) - \mu_Y^2$

$$\sigma_X^2 = (0 + 1/2) - 1/4 = 1/4$$

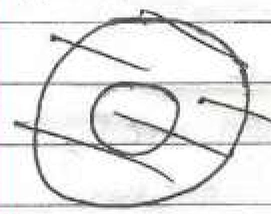
$$\sigma_Y^2 = (0 + 3/8 + 3/2 + 9/8) - (9/4)$$

$= i f(a) [\theta]_0^{2\pi}$

$= i f(a) (2\pi - 0)$

$\boxed{I = 2\pi i f(a)}$

$\boxed{\oint_C \frac{f(z)}{(z-a)} dz = 2\pi i f(a)}$ hence proved



* Generalized Cauchy's integral formula

If $f(z)$ is analytic inside and on a simple closed curve C & if a is a point within C then,

$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz$$

Proof: we have Cauchy's integral formula,

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a} dz$$

$$f'(a) = \frac{1}{2\pi i} \oint_C f(z) \frac{\partial}{\partial a}\left[\frac{1}{z-a}\right] dz$$

$$f'(a) = \frac{1}{2\pi i} \oint_C f(z) \{(-1)(z-a)^{-2} \cdot (-1)\} dz$$

$$f'(a) = \frac{1!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz$$

$$f''(a) = \frac{1!}{2\pi i} \oint_C f(z) \frac{\partial}{\partial a}[(z-a)^{-2}] dz$$

$$f''(a) = \frac{2!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^3} dz$$

$\vdots$

$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz$$

$f^{(n)}$ denotes the n^{th} derivative of $f(z)$ at $z = a$.

9) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where C is circle $|z| = 3$

Sol: By partial fractions, $\frac{1}{(z-1)^2(z-2)}$

Let, $\frac{1}{(z-1)^2(z-2)} = \frac{A}{(z-1)} + \frac{B}{(z-1)^2} + \frac{C}{(z-2)}$ —(1)

$1 = A(z-1)(z-2) + B(z-2) + C(z-1)^2$

$z = 1$; $1 = B(-1) \Rightarrow \boxed{B = -1}$

$z = 2$; $1 = C(1) \Rightarrow \boxed{C = 1}$

equating the coefficient of z^2 on both sides we have,

$0 = A + C$ or $A = -C \Rightarrow \boxed{A = -1}$

let, $f(z) = \sin \pi z^2 + \cos \pi z^2$

multiplying (1) by $f(z)$ and integrating wrt z over C by using the value of the constant obtained we have,

$$I = \oint_C \frac{f(z)}{(z-1)^2(z-2)} dz$$

$$= -\oint_C \frac{f(z)}{z-1} dz - \oint_C \frac{f(z)}{(z-1)^2} dz + \oint_C \frac{f(z)}{(z-2)} dz$$

$I = I_1 + I_2 + I_3$

$C: |z| = 3$

the points $z = 1$ & $z = 2$ both lie within C

we evaluate I_1 & I_3 by Cauchy's integral formula & I_2 using its generalised form by taking $n = 1$.

$I_1 = -[2\pi i f(1)] = -2\pi i(\sin\pi + \cos\pi)$
$= -2\pi i(0-1) = 2\pi i$
$I_2 = -[2\pi i f'(1)]$ but $f'(z) = 2\pi z(\cos\pi z^2 - \sin\pi z^2)$
$I_2 = -[2\pi i \cdot 2\pi(\cos\pi - \sin\pi)] = 4\pi^2 i$
$I_3 = 2\pi i f(2) = 2\pi i(\sin 4\pi + \cos 4\pi)$
$= 2\pi i(0+1) = 2\pi i$
$\therefore I = 2\pi i + 4\pi^2 i + 2\pi i$
$= 4\pi i + 4\pi^2 i$
$\boxed{I = 4\pi i(1+\pi)}$ where $C: |z| = 3$

10) Evaluate $\int_C \frac{z e^z}{(z^2-1)} dz$ where $C: |z| = 2$

Sol: $(z^2-1) \Rightarrow (z^2-1^2) \Rightarrow (z-1)(z+1)$

$\therefore \int_C \frac{z e^z}{(z-1)(z+1)} dz$

by partial fractions,

$\frac{1}{(z-1)(z+1)} = \frac{A}{(z-1)} + \frac{B}{(z+1)}$

$1 = A(z+1) + B(z-1)$

put $z=1$

$1 = A(1+1) + \cancel{B(1-1)}^{0} \Rightarrow 1 = 2A \Rightarrow \boxed{A = \frac{1}{2}}$

put $z = -1$, $1 = \cancel{A(-1+1)}^{0} + B(-1-1)$
$1 = -2B \Rightarrow \boxed{B = \frac{-1}{2}}$

$\therefore \frac{1}{(z-1)(z+1)} = \frac{1}{2}\frac{1}{(z-1)} - \frac{1}{2}\frac{1}{(z+1)}$

$\therefore$ The points, $z = a = +1$ &
$z = a = -1$

$\int_C \frac{z e^z}{(z-1)(z+1)} dz = \frac{1}{2}\int_C \frac{z e^z}{(z-1)} dz - \frac{1}{2}\int_C \frac{z e^z}{(z+1)} dz$

we have by Cauchy's integral formula,

$\int_C \frac{f(z)}{z-a} dz = 2\pi i f(a)$

where $f(z) = z e^z$ & $a = 1, -1$

$f(1) = 1e^1 = 2.7183$
$f(-1) = -1e^{-1} = -0.3679$

$\int_C \frac{z e^z}{(z-1)} dz = 2\pi i f(1) = 2\pi i(2.7183)$
$= 5.4366\pi i$

$\int_C \frac{z e^z}{(z+1)} dz = 2\pi i f(-1) = 2\pi i(-0.3679)$
$= -0.7358\pi i$

$\Rightarrow \int_C \frac{z e^z}{(z-1)(z+1)} dz = \frac{1}{2}[5.43\pi i] - \frac{1}{2}[-0.735\pi i]$

$= \frac{\pi i}{2}[5.43 + 0.73]$

$= \frac{77}{25}\pi i$

$\therefore \boxed{\int_C \frac{z e^z}{(z-1)(z+1)} dz = \frac{77}{25}\pi i}$

1AM19IS058

If YOU WANT to SHINE like a SUN. FIRST BURN LIKE a SUN

Inspirational Quotes by

-A.P.J Abdul Kalam

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Subject Chemistry Assignment

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Electro Chemistry And Energy Storage Systems

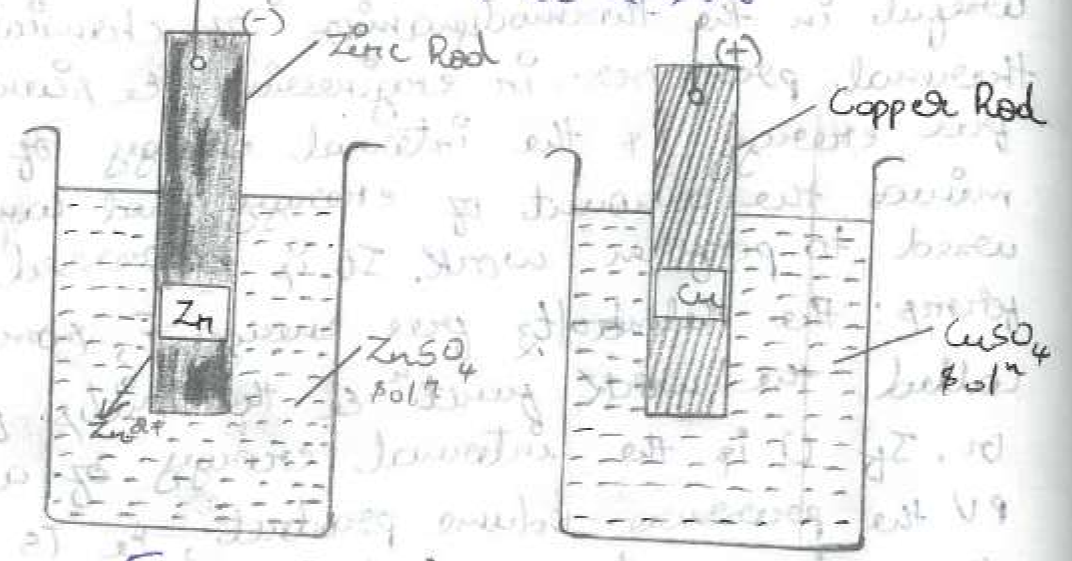
① Define the following terms? ⓐ Free energy ⓑ cell Potential ⓒ single electrode Potential.

⇒ Free energy : "The thermodynamic free energy is the amount of work that a thermodynamic system can perform". The concept is useful in the thermodynamics of chemical or thermal processes in engineering & science. The free energy is the internal energy of a system minus the amount of energy that cannot be used to perform work. It is expressed in two forms: the Helmholtz free energy F, sometimes called the work funct^n & the Gibbs free energy G. If U is the internal energy of a system, PV the pressure-volume product, & TS the temperature-entropy-product then,

$F = U - TS$ & $G = H - TS$, where H is the enthalpy. ($H = U + PV$, where P is the pressure & V is the Volume.) Therefore $G = U + PV - TS$. Both enthalpy & Gibbs free energy are measured in $KJ\ mol^{-1}$.

Cell emf (Potential) : "This difference in potential b/w the electrodes is known as electromotive force (emf) of the cell. E_{cell} for an electrochemical cell may be defined as the difference in potential which causes electrons to flow from one electrode to another". It depends on nature of the electrode, temperature, conc^n etc.

Single electrode Potential (E): The potential developed metal & the sol^n, when it is in contact with a solution of its own ions. It is denoted by E. Since electrode potential depends on conc^n, temperature etc, electrode potential indicates a measure of the tendency of an electrode to gain electrons i.e. to undergo reduct^n & it is represented by the following react^n $M^{n+} + ne^- \leftrightarrow M$



$E_{cell} = E_{cathode} - E_{anode}$

Electrode Potential

② What is Nernst eq^n? Derive Nernst eq^n thermodynamically & mention its applications.

=> "Nernst equation is a quantitative relationship between electrode & concentration of the electrolyte species involved.

Consider the following reversible electrode reaction

$M^{n+} + ne^- \leftrightarrow M$ — ①

The decrease in free energy is the maximum amount of work that can be obtained from a chemical reaction.

$-\Delta G = W_{max}$ — ②

W_{max} for an electrochemical cell is given by the eq^n

$\Delta G = -nFE$ — ③

where n is the no. of electrons transferred in the reaction & E is the electrode potential of the electrode in Volts.

Under standard conditions, when the concentration of all species is unity, the standard free energy change ΔG° is given by

$\Delta G^\circ = -nFE^\circ$ — ④

Where E° is the standard electrode potential, the decrease in free energy (ΔG), accompanying the process in eq^n 1 is given by

$\Delta G = \Delta G^\circ + RT \ln K_c$ — ⑤ where $K_c = [M]/[M^{n+}]$

$\Delta G = \Delta G^\circ + RT \ln [M]/[M^{n+}]$ — ⑥

Substituting ΔG & ΔG° from eq^n ③ & ④ in ⑥ we get,

$-nFE = -nFE^\circ + RT \ln [M]/[M^{n+}]$ — ⑦ dividing by $-nF$

$E = E^\circ - (RT/nF) \ln [M]/[M^{n+}]$ — ⑧ since $[M] = 1$

$E = E^\circ + (RT/nF) \ln [M^{n+}]$

$E = E^\circ + (2.303\, RT/nF) \log [M^{n+}]$

where E° is the standard electrode potential, R is the gas constant, 'T' is the absolute temperature & F is faraday. At 298 K, substitute the values for $R = 8.314\ JK^{-1} mol^{-1}$, $F = 96500\ C\,mol^{-1}$ the Nernst eq^n reduces to

$E = E^\circ + 0.0591/n \log [M^{n+}]$ — ⑨ (eq^n ⑨ is the Nernst eq^n for electrode potential of a single electrode).

Applications :

(i) The equation is used to determine the electrode potential of an electrode if the concentration is known.

(ii) The equation can be used to calculate the emf of a cell. consider the cell reaction $aA + bB + cC + dD$. The Nernst equation for the emf of the cell at 298 K is, $E_{cell} = E^\circ_{cell} - 0.0591/n \cdot \log [C]^c [D]^d / [A]^a [B]^b$

where n is the no. of electrons transferred during the cell reaction & E°_{cell} is the standard emf of the cell.

---

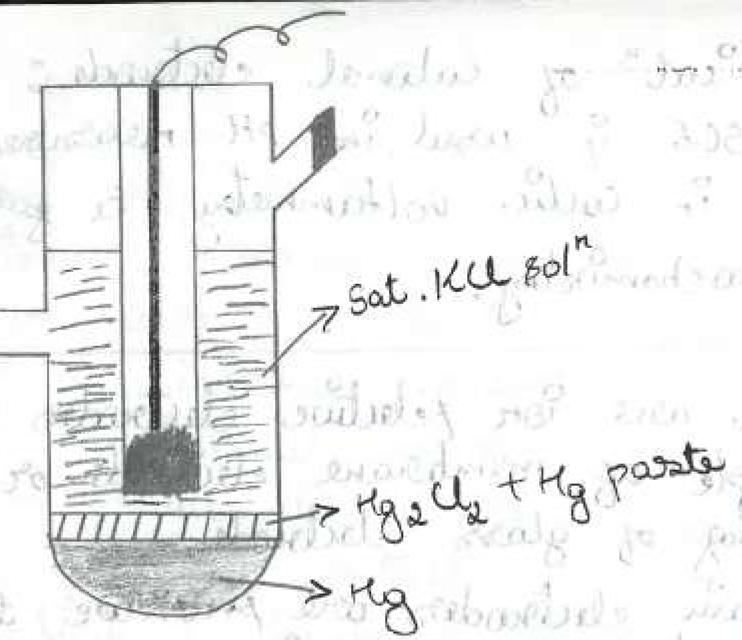
(3) what are reference electrodes? Explain the construction & working of glass calomel electrode mentioning its applications.

=> "Electrodes with a well known & stable E° values are known as reference electrodes."

Construction & working of calomel electrode :

It is a metal-insoluble salt electrode consisting of mercury, mercurous chloride & a solution of KCl. Mercury is placed at the bottom of a glass tube having a side tube on each side. Mercury is covered with a paste of mercurous chloride (calomel) with mercury & KCl sol^n. KCl sol^n is introduced above the paste by the side tube. Pt wire sealed into glass tube is dipped into mercury & used to provide the external contact; the conc. of KCl used is either deci-normal, normal or saturated solution.

The electrode representat^n is : $Hg / Hg_2Cl_2 / Cl^-$



Calomel electrode

Electrode reactions : when it acts as anode the electrode reaction is, $2Hg \longrightarrow Hg_2^{2+} + 2e^-$

$$Hg_2^{2+} + 2Cl^- \longrightarrow Hg_2Cl_2$$

overall react^n : $2Hg + 2Cl^- \longrightarrow Hg_2Cl_2 + 2e^-$

when it acts as cathode the electrode react^n is,

$$Hg_2^{2+} + 2e^- \longrightarrow 2Hg$$

$$Hg_2Cl_2 \longrightarrow Hg_2^{2+} + 2Cl^-$$

overall react^n : $Hg_2Cl_2 + 2e^- \longrightarrow 2Hg + 2Cl^-$

The net reversible electrode react^n is

$$Hg_2Cl_{2(s)} + 2e^- \longleftrightarrow 2Hg(l) + 2Cl^-$$

A/C to Nernst eq^n, the potential of calomel electrode at 298 K is related to Cl^- ions as,

$E_{cal} = E^\circ_{cal} - 0.0591/n \log [Cl^-]^2$ where $n = 2$

$E_{cal} = E^\circ_{cal} - 0.0591 \log [Cl^-]$

The potential values of calomel electrode of different conc. of KCl measured using SHE are;

$0.1N\ KCl = 0.3338\ V$;

$1.0N\ KCl = 0.282\ V$.

Applicat^n of calomel electrode:

(i) The SCE is used in pH measurement.

(ii) Used in cyclic voltammetry & general aqueous electrochemistry.

---

(4) What are ion selective electrodes? Explain the Principle of membrane electrode or explain the working of glass electrode.

⇒ "Certain electrodes are sensitive to only certain ions when they are in contact with different kinds of ions. Electrodes with specific ability to respond only certain ions present in sol^n & developing a potential with respect to that ions while ignoring other ions are called as ION selective electrodes." Since membrane is the most important component of ion selective electrodes hence, these electrodes are also known as "membrane electrodes".

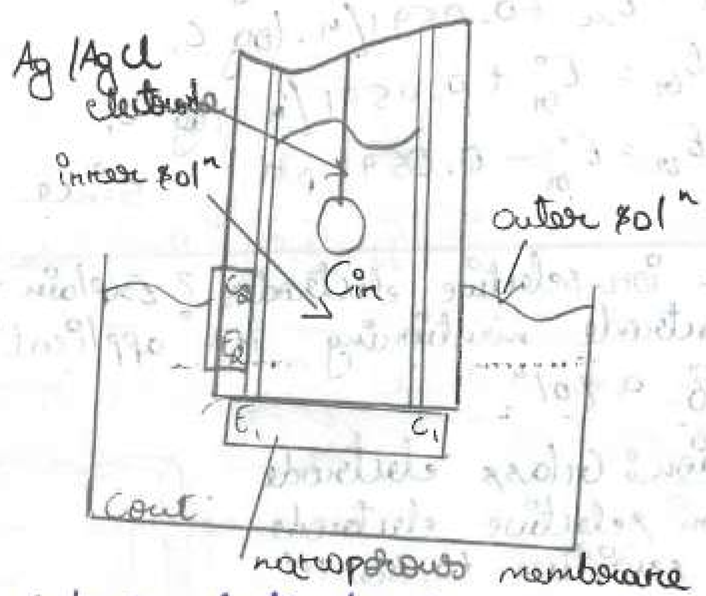
Principle of membrane electrode or working of Glass electrode:

When two sol^n containing the same ions (to which the membrane is sensitive to) but of different concn are seperated by a membrane, across the membrane there arises a potential due to difference in conc^n of the species. The potential so developed is called boundary potential.

If the concn of one sol^n is known the concnt^n of the other sol^n can be calculated. The potential so developed is a measure of concnt^n of species of the interest.

The electrode generally consists of a membrane which is capable of exchanging the specific ions with which it is in contact. An ion selective electrode consists of an ion selective membrane in contact with an analyte sol^n on one side & internal reference sol^n on the other side. An internal reference electrode is constituted in contact with the reference sol^n to provide electrical contact.

A potential is developed in the internal region of the membrane & also on the outer region as shown in figure.



Glass electrode immersed in a sol^n

The potential which is developed across the membrane is given by, $E_j = E_1 - E_2$ —— (1)

where E_1 is the potential developed at the outer surface of the membrane & E_2 is the potential developed at the inner surface of the membrane.

By Nernst eq^n (i) becomes: $E_j = E^\circ + 0.0591/n . \log C_1 - (E^\circ + 0.0591/n . \log C_2)$

$E_j = 0.0591/n . \log C_1/C_2$ —— (2)

where C_1 is the conc. of the ions in the analyte sol^n & C_2 is the conc. of ions in the internal reference sol^n since C_2 is known

$E_j = 0.0591/n \log C_1 - 0.0591/n \log C_2$

$E_j = K + 0.0591/n \log C_1$ —— (3) (where $K = -0.0591/n \log C_2$)

The overall potential of the membrane electrode is given by $E_M = E_j + E_{ref}$

$E_M = E_j + E_{ref}$ —— (4)

$= K + 0.0591/n \log C_1 + E_{ref}$ (from eq^n (3))

$= E^\circ_M + 0.0591/n \log C_1$

At 298 K, $E_M = E^\circ_M + 0.0591/n \log C_1$, where $E^\circ_M = K + E_{ref}$

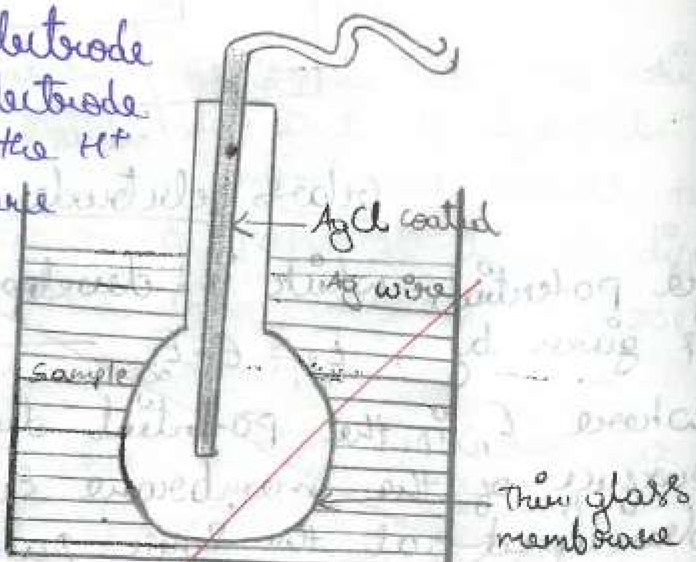
or $E_G = E^\circ_G + 0.0591/n \log C_1$

or $E_G = E^\circ_G - 0.0591\, pH$ since $pH = -\log C_1$

---

(5) What are ion selective electrodes? Explain the construct^n of glass electrode mentioning its applicat^n in determining the pH of a sol^n.

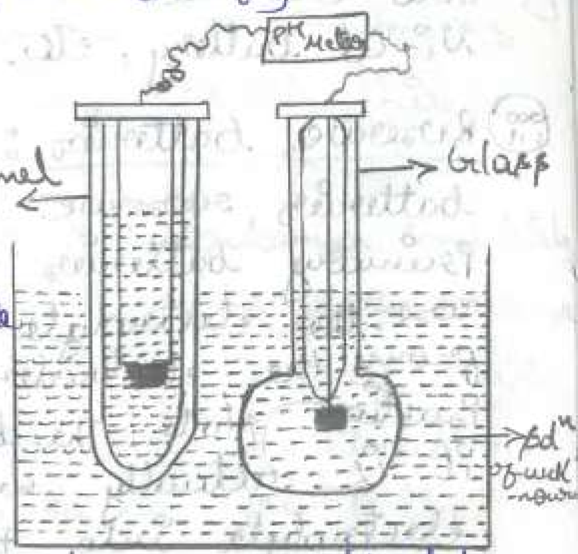
⇒ Construction: Glass electrode is an ion selective electrode which is sensitive to the H^+ ions in the sol^n & hence is widely used for pH determinat^n.



The glass electrode consists of a glass bulb made up of a special type of glass with high electrical conductance. The glass bulb filled with a sol^n of constant pH (0.1 M HCl) & has a Ag-AgCl electrode, which is the internal reference electrode & also serves for the external contact. The electrode is dipped in a sol^n containing H^+ ions. Th

Ag | AgCl | 0.1 M HCl | glass | HCl (pH=?)

To determine pH of a given sol^n, the glass electrode is dipped in a sol^n whose pH needs to be determined. It is combined with a saturated calomel electrode. The cell notation is written as:



$Hg \mid Hg_2Cl_2 \mid Cl^- \parallel$ sol^n (pH=?) | glass | 0.1 M HCl | AgCl | Ag

The emf of the cell so formed is determined potentiometrically.

$E_{cell} = E_c - E_a$

$E_{cell} = E_G - E_{cal}$

$E_{cell} = E^\circ_G - 0.0591\, pH - E_{cal}$ (from the working of glass electrode)

$pH = (E^\circ_G - E_{cal} - E_{cell}) / 0.0591$

---

(6) Define Primary, secondary & reserve batteries.

⇒ (i) Primary batteries: In these cells electrical energy is produced at the expense of chemical energy as long as active materials are present. They cannot be recharged & reused. Ex: Dry cell (Zn-MnO_2 battery).

(ii) Secondary batteries: These are also called as storage cells. Electrical energy is stored as chemical energy. Once used, they can be recharged by passing current & can be reused.

A primary cell acts as only as a galvanic cell or voltaic cell, a secondary cell can act both as galvanic cell & electrolytic cell. During discharge it acts as a galvanic cell, converting chemical energy into electrical energy & during charge it acts as an into chemical energy. Ex: Lead storage battery, Ni-Cd battery, etc.

(iii) Reserve batteries: Also called deferred-action batteries, reserve batteries are special purpose primary batteries usually designed for emergency use. The electrolyte is usually stored separately from the electrodes which remain in a dry inactive state. The battery is only activated when it is actually needed by introducing the electrolyte into the active cell area. This has the double benefit of avoiding deterioration of the active materials during storage & at the same time it eliminates the loss of capacity due to self discharge until the battery is called into use. They can be stored for more than 10 years. Ex: Ampoule batteries, Thermal batteries & water-activated batteries.

---

(7) Explain the construct^n & working of Li-ion batteries, mention its applicat^n.

=> Construction: Anode - Lithium intercalated graphite or polymer

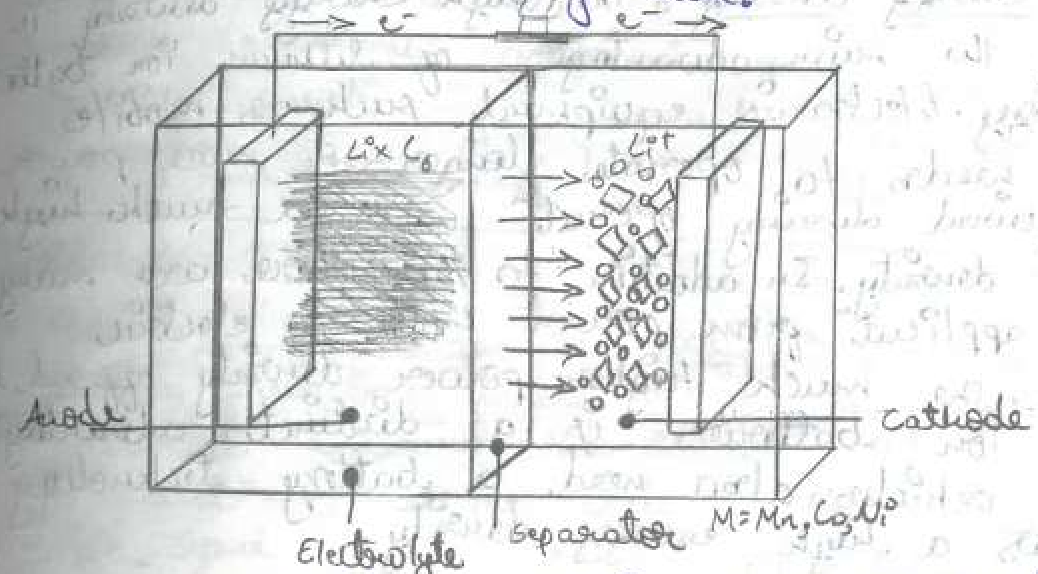
Cathode: $LiCoO_2$ or $LiMn_2O_4$

Electrolyte: Lithium salts like $LiPF_6$ in an organic solvent.

Separator: Microporous polyethylene or polypropylene

Solvent: Diethyl carbonate, dimethyl carbonate etc.

Working: In lithium ion battery, lithium ions move between the electrodes, using an intercalated electrode material. Metal atoms & ions can enter layered solids reversibly hence this process is referred to as intercalation. During discharge, Li comes out of the layers of $LixC_6$, as Li^+ ions & migrates towards the cathode & gets discharged. Cobalt oxide being a layered solid accommodates Li in its layers thus completing the conduction process. This process continues as long as the Li present in the graphite & once the Li is consumed completely the battery needs charging. During charge, the reverse react^n takes place where in Li in the cobalt oxide is released as Li^+ ions & migrates back to the layers of graphite. Thus, the Li ion battery acts as a secondary cell.



Electron & Li-ion move reversely at charging.

Electrode react^n: During discharge

At anode: $LixC_6 \longrightarrow xLi^+ + xe^- + 6C$ —(1)

At cathode: $Li_{1-x}CoO_2 + xLi^+ + xe^- \longrightarrow LiCoO_2$ —(2)

Net cell react^n is (adding (1) & (2) eq^n)

$LixC_6 + Li_{1-x}CoO_2 \longrightarrow 6C + LiCoO_2$

During charge, the above react^n are reversed

The net reversible react^n is

$LixC_6 + Li_{1-x}CoO_2 \longleftrightarrow 6C + LiCoO_2$

Applicat^n of Lithium-ion Batteries:

Li-ion batteries are available in all shapes & sizes. Some of the most common applicat^n of lithium-ion batteries are:

* Power backups / UPS.
* Mobile, Laptops, & other commonly used consumer electronic goods.
* Electric mobility.

(8) What are the advantages of Li-ion battery as electrochemical energy system for electric vehicles?

=> There are many advantages of Li-ion battery as electrochemical energy system for electric vehicles. The advantages include:

* High energy density: The high energy density is one of the main advantages of lithium ion battery technology. Electronic equipment such as mobile phones needs to operate longer & more power is required during operatn with a much higher energy density. In addition to this, there are many power applicatn from power tools to electric vehicles. The much higher power density offered by lithium ion batteries is a distinct advantage. Electric vehicles also need a battery technology that has a high energy density.

* Self-discharge: Lithium ion cells have low self-discharged than that of other rechargeable cells such as Ni-Cad & Ni-MH batteries.

* Low maintenance: They do not require maintenance to ensure their performance.

* Cell voltage: The voltage produced by each lithium ion cell is about 3.6 volts. This has many advantages as the voltage of each lithium ion cell is higher, requires less cells in many battery applications. For smart phones a single cell is sufficient & this simplifies the power management.

* Load characteristics: The load characteristics of a lithium ion cell or battery are reasonably good. They provide a reasonably constant 3.6 volts

* No requirement for priming: Some rechargeable cells need to be primed when they receive their first charge. One advantage of lithium ion batteries is that there is no requirements for this they are supplied operational & ready to use.

* Variety of types: There are several types of lithium ion cell available. This advantage of lithium ion batteries can mean that the right technology can be used for a particular applicatn needed. Some forms of lithium ion battery provide a high current density & are ideal for consumer mobile electronic equipment. Others are able to provide much higher current levels & are ideal for power tools & electric vehicles.

(9) Explain recycling of lithium ion batteries?

=> Recycling of lithium ion batteries: Widespread battery recycling would avoid the hazardous material entering the environment during it's productn & at the end of the battery's useful life. Some of the recycling processes are,

(i) Smelting: This process recover basic elements or salts. These processes are operational currently on a large scale & can accept multiple kinds of batteries, including lithium ion & nickel-metal hydride. Smelting takes place at high temperatures, & organic materials, including the electrolyte & carbon anodes, are burnt as fuel or reductant. The valuable metals are recovered & sent to refining so that the product is suitable for any use. The other materials, including lithium, are contained in the slag, which is used as an additive in concrete.

(ii) Direct recovery: At the other extreme, some recycling processes directly recover battery-grade

$E^\circ_{cell} = E^\circ_C - E^\circ_A$
$= 0.80 - (-0.42)$
$E^\circ_{cell} = 1.22V$

$E_{cell} = 1.22 - \frac{0.0591}{2} \log \frac{[-0.42]}{[0.80]^2}$
$= 1.22 - 0.02955 [\log[0.42] - 2\log[0.80]]$
$E_{cell} = 1.21V$

(14) write a note on sodium ion batteries?
⇒ A sodium-Ion (Na-Ion) Battery system is an energy storage system based on electrochemical charge ~~Reactio~~ Reactions that occur b/w a +ve electrode (cathode) composed of sodium-containing layered materials & a -ve electrode (anode) that is typically made of hard carbons or intercalat^n compounds. The electrodes are separated by some porous material which allow ionic flow b/w them & are immersed in an electrolyte that can be made up of either aqeous sol^n (such as Na_2SO_4) or non-aqeous sol^n (ex: salts in propylene carbonate). When the battery is being charged, Na atoms in the cathode release electrons to the external circuit & become ions which migrate through the electrolyte toward the anode, where they combine with electrons from the external circuit while reacting with the layered anode material. This process is reversed during discharge.

5/3/20

Assignment 02

(1) what are the specifications of drinking water as specified by WHO?

⇒ WHO produces international norms on water quality & human health in the form of guidelines that are used as the basis for regulation & standard setting, in developing & ~~standard~~ developed countries world wide. The quality of drinking water is a powerful environmental determinant of health. Assurance of drinking water safety is a foundation for the prevention & control of water bone diseases. The guidelines developed by WHO are prepared through a vast global consultative process involving WHO member state), national agencies in consultat^n with the WHO. Expert advisory panel. Specifications of drinking water as per WHO standards are given below:

Parameters	WHO
pH	6.9 - 9.2
TDS	500 - 1500 mg/l
EC	300 umhos/cm
Cl^-	200 - 600 mg/l
SO_4^{2-}	200 - 250 mg/l
NO_3^-	40 - 50 mg/l
F^-	1 - 1.5 ppm
Ca^{2+}	75 - 200 mg/l
Mg^{2+}	30 - 150 mg/l
Na^+	50 - 60 mg/l
K^+	20 mg/l
TH	100 - 500 mg/l

② Explain the following concentration terms:

ⓐ Normality (N): Normality of a solⁿ is defined as the no. of gram equivalents of solute per litre of the solⁿ.

Normality = No. of gram equivalent × [volume of solⁿ in litres]$^{-1}$

ⓑ Molarity (M): Molarity of a solⁿ is defined as the no. of gram moles of solute per litre of the solⁿ.

$M = \frac{n}{V}$

ⓒ Parts per million (PPm): It is defined as the no. of parts by mass of the solute per million parts by mass of solⁿ.

$PPm = \frac{\text{mass of solute}}{\text{mass of sol}^n} \times 10^6$

The term PPm is used when the amount of a substance in the solⁿ is small.

ⓓ Mole fraction: The mole fraction of a solute is the ratio of the no. of moles of the solute to the total no. of moles of both the solute & the solvent.

$X_{solute} = \frac{\text{Moles of solute}}{\text{Total moles of sol}^n}$

$X_{solvent} = \frac{\text{moles of solvent}}{\text{Total moles of sol}^n}$

$X_{solute} + X_{solvent} = 1$

③ Explain the determination of total hardness by EDTA complexometric method?

⇒ Principle: The determination of hardness is carried out by titrating water sample with sodium salt of Ethylene diamine tetra Acetic acid (EDTA) using Eriochrome black-T as an indicator & keeping the pH of the water at 9.0–10.0. The end point is the change in colour from wine-red to clear blue, when the EDTA solⁿ complexes the calcium & magnesium salt completely.

$HOOCH_2C$ \
$\quad > N-CH_2-CH_2-N <$ CH_2COOH
$HOOCH_2C$ / $\quad$ CH_2COOH

EDTA (H_4Y)

$NaOCOCH_2$ \
$\quad NCH_2CH_2N <$ CH_2COONa
$HOCOCH_2$ / $\quad$ CH_2COOH

Na_2H_2Y

$[Ca^{2+}$ or $Mg^{2+}]$ + EBT ⟶ [Ca-EBT] or [Mg-EBT]
Hardness-salts $\quad$ indicator $\quad$ unstable complex (wine red)

[Ca-EBT] or [Mg-EBT] + EDTA ⟶ [Ca-EDTA] or [Mg-EDTA] + EBT (clear blue).
unstable complex (wine red) $\quad$ Stable complex (colourless)

Chemical required:

(i) Preparation of standard hard water (0.01M): Dissolve 1g of pure, dry $CaCO_3$ in minm quantity of dil. HCl & then evaporate the solⁿ to dryness on a water bath. Dissolve the solⁿ to dryness on a water bath. Dissolve the residue in distilled water to make 1 litre solⁿ. Each ml of this solⁿ thus contains 1mg of $CaCO_3$ equivalent hardness.

(ii) Preparation of EDTA solⁿ: Dissolve 4g of pure EDTA crystals + 0.1g $MgCl_2$ in 1 litre of distilled water.

(iii) Preparation of indicator (EBT): Dissolve 0.5g of Eriochrome Black-T in 100ml alcohol.

(iv) Preparatⁿ of Buffer solⁿ: Add 67.5g of ~~Eriochrome~~ ~~NH4~~ NH_4Cl to 570 ml of Conc. Ammonia solⁿ & then dilute with distilled water to 1 litre.

through a transparent medium, the rate of decrease in intensity dI with the thickness of the medium dt is proportional to the intensity of the light I.

$$\therefore -\frac{dI}{dt} = KI$$

where, K is a proportionality constant. Rewriting the above eq^n, $-\frac{dI}{I} = K\,dt$

Integrating this eq^n & substituting $I = I_0$ when $t = 0$

$$\ln \frac{I_0}{I_t} = Kt$$

$$I_t = I_0 e^{-Kt} \quad \text{--- (2)}$$

Thus there is an exponential decrease in intensity of transmitted light with the increase in thickness of the medium.

Beer's law: It states that the intensity of the transmitted light decreases exponentially as the conc^n of the medium increases arithmetically.

$$I_t = I_0 e^{-Kc} \quad \text{--- (3)}$$

where, c is the molar conc^n of the sample sol^n.

Combining eq^n (2) & (3), $I_t = I_0 e^{-Kct}$ --- (4)

where ϵ is called molar absorptivity. It is a constant for a given substance at a given wave length. If c is expressed in mol dm^{-3} & t in cm, ϵ has the unit dm^3 mol^{-1} cm^{-1}. eq^n (4) can be written as, $\log \frac{I_0}{I_t} = \epsilon c t$ --- (5)

eq^n (5) is referred to as Beer-Lambert's law. Beer Lambert's law is the basis for the colorimetric method of analysis.

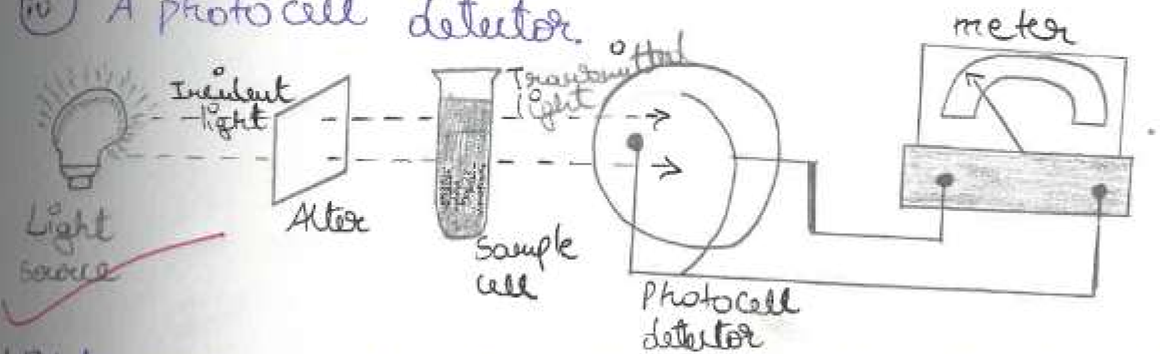
The term $\log \frac{I_0}{I_t}$ is called the optical density or absorbance A & I_t/I_0 is the transmittance T. The relat^n b/w A, t, c & ϵ is given by the eq^n,

$$A = \epsilon c t = \log \frac{I_0}{I_t} = \log \frac{1}{T} = -\log T$$

If the path length of the cell is kept constant, then absorbance A is proportional to the conc^n c.

(1) Explain the instrumentation & any 2 applications of colorimetry.

⇒ Instrumentation: The instrument used to measure the absorbance of a sol^n is called photoelectric colorimeter. The photoelectric colorimeter consists of: (i) Tungsten lamp as the light source.
(ii) A filter which provides the desired wavelength range where in the sol^n gives the max^m absorbance.
(iii) A sample cell.
(iv) A photocell detector.



Light from a tungsten lamp is allowed to fall on the sol^n taken in the sample cell after passing through the filter. First, a blank sol^n is taken in the sample cell & placed in the path of the light beam. Its absorbance is adjusted to zero (or transmittance to 100) on the meter. Next the analyte sol^n is placed in the path of the light beam & the quantity of light absorbed is measured as its absorbance (or transmittance).

Applications of Calorimetry:

Colorimetry is a versatile method of determining the conc^n of metals & non-metals present in small quantities in ores, soil samples & alloys. The sample is dissolved in a suitable acid & a known amount of the sol^n is treated with a reagent that produces a characteristic is measured at an optimum wavelength (λ_{max}) using a photoelectric calorimeter. This is illustrated for the analysis of copper in a steel sample dissolved in hydrochloric acid

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